CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

27 JUNE 2008 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Write your solutions, with full working, in the answer booklet provided.

4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this question.

1. (a) Given that $x = \ln [y + \sqrt{(y^2 - 1)}]$, $y > 1$, express y in terms of x. **[5 marks]**

   (b) Use the substitution $u = \sin x$ to find

   $$\int \cos^3 x \, dx.$$ **[6 marks]**

   (c) Engine oil at temperature T °C cools according to the model

   $$T = 60\, e^{-kt} + 10$$

   where t is the time in minutes from the moment the engine is switched off.

   (i) Determine the initial temperature of the oil when the engine is **first** switched off. **[2 marks]**

   (ii) If the oil cools to 32°C after three minutes, determine how long it will take for the oil to cool to a temperature of 15°C. **[7 marks]**

   Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) (i) Write the general term of the series whose first four terms are

   $$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \frac{1}{7 \times 9} + \dots$$ **[2 marks]**

   (ii) Use the method of differences to find the sum of the first n terms. **[5 marks]**

   (iii) Show that the series converges and find its sum to infinity. **[3 marks]**
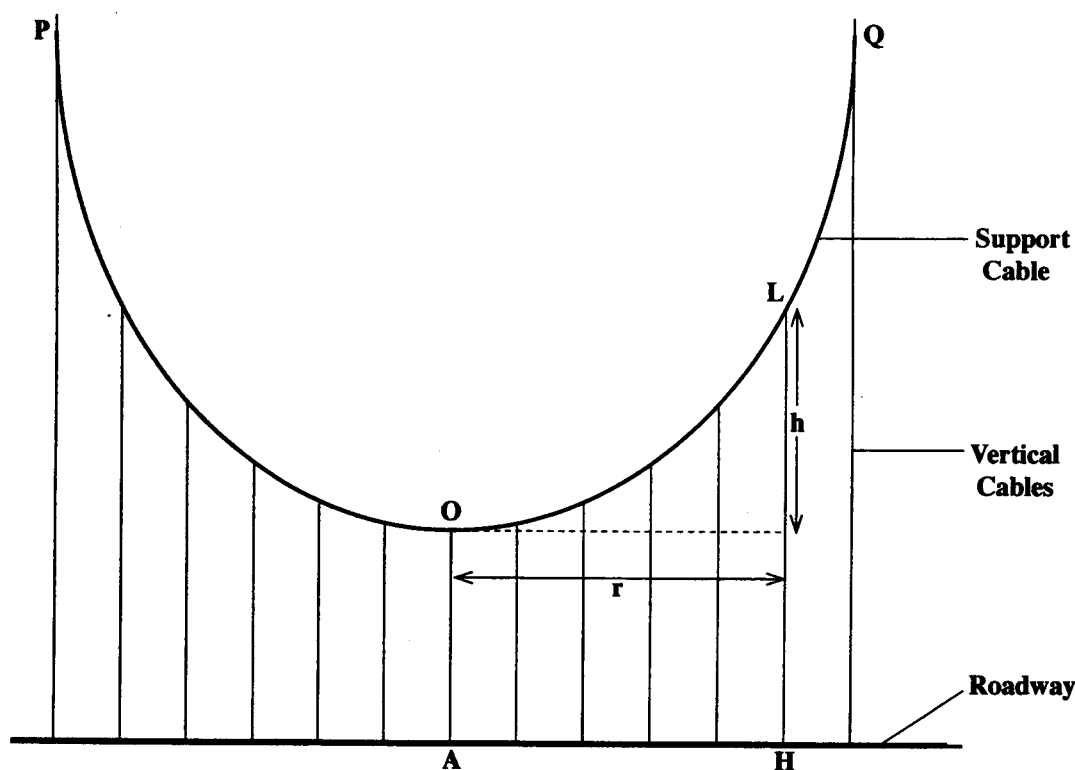
(b)   The diagram below **(not drawn to scale)** shows part of the suspension of a bridge. A support cable POQ, is in the shape of a curve with equation

$$y = \frac{1}{10} \left| x^{3/2} \right| + c, \text{ where } c \text{ is a constant.}$$

Starting at P, through O and finishing at Q, 51 vertical cables are bolted 1 metre apart to the roadway and to the support cable POQ. The shortest vertical cable OA has a length of 5 metres, where O is the lowest point of the support cable.

The cost, in dollars, of installing the cable LH at a horizontal distance of r metres from OA is $100 plus $ $h \sqrt{r}$, where h is the height of the point L above O.

(i)   Find, in terms of *r*, the cost of installing the cable LH.   **[4 marks]**

(ii)   Hence, obtain the **total** cost of installing the 51 vertical cables.   **[6 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this question.

3.   (a)   Let $z = \dfrac{(1 - 2i)\,(7 + i)}{(1 + i)^2}$.

      (i)   Express z in the form $a + bi$, where $a, b \in \mathbf{R}$.   **[5 marks]**

      (ii)   Calculate the **exact** value of $|z|$.   **[3 marks]**

  (b)   Two 3 x 1 matrices **X** and **Y** satisfy the equation **X = AY**, where the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 3 & 2 & 4 \\ 4 & 1 & 6 \end{pmatrix} \quad \text{is non-singular.}$$

Find

      (i) $\mathbf{A}^{-1}$ **[8 marks]**

      (ii)   **Y**, when $\mathbf{X} = \begin{pmatrix} 6 \\ 4 \\ 11 \end{pmatrix}$.   **[4 marks]**

Total 20 marks

END OF TEST