TEST CODE **02234010/SPEC**

FORM 02234010/SPEC2007

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

Paper 01

90 minutes

READ THE FOLLOWING DIRECTIONS CAREFULLY

1. In addition to this test booklet, you should have an answer sheet.

2. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.

3. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The expression $(1 + \sqrt{3})^2$ is equivalent to

   (A) 4
   (B) 10
   (C) $1 + 3\sqrt{3}$
   (D) $4 + 2\sqrt{3}$

   Sample Answer

   Ⓐ Ⓑ Ⓒ ●

   The best answer to this item is "$4 + 2\sqrt{3}$", so answer space (D) has been blackened.

4. If you want to change your answer, be sure to erase your old answer completely and fill in your new choice.

5. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. You can come back to the harder item later.

6. You may do any rough work in this booklet.

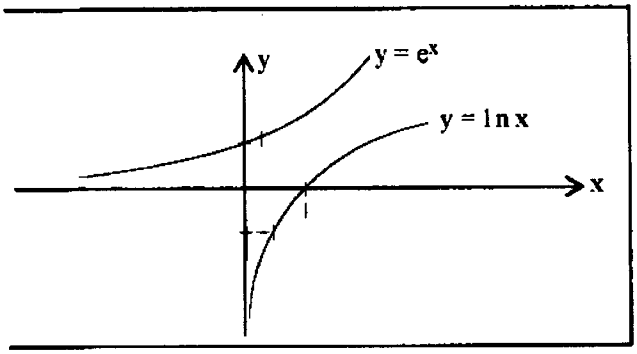
7. The use of non-programmable calculators is allowed.

8. This test consists of 45 items. You will have 90 minutes to answer them.

9. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

<u>Item 1</u> refers to the following graph.

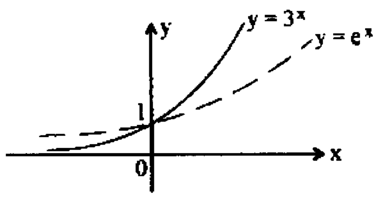


1. The graphs of $y = e^x$ and $y = \ln x$ in the diagram above are reflections of each other in the line

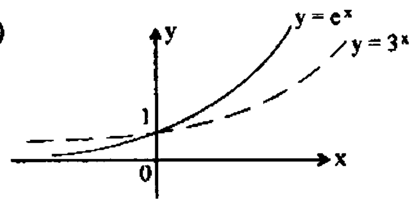
(A) $y = 0$
(B) $x = 0$
(C) $y = -x$
(D) $y = x$

2. Which of the graphs in the diagrams below represents $y = 3^x$ and $y = e^x$?

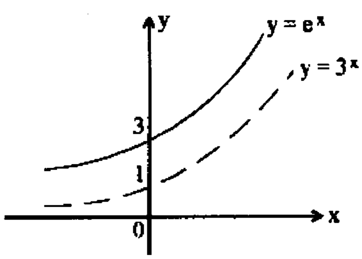
(A)



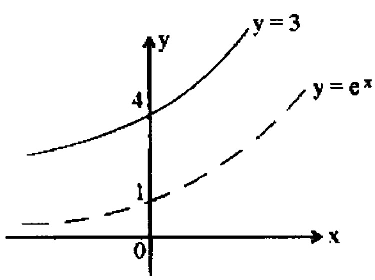
(B)



(C)



(D)



3. $\frac{1}{2}\ln 4 - \frac{1}{2}\ln 3$ can also be expressed as

I. $\frac{1}{2}\ln 1$

II. $\frac{1}{2}\ln\frac{4}{3}$

III. $\ln 2 - \ln\sqrt{3}$

IV. $\ln\frac{2}{\sqrt{3}}$

(A) I and II only
(B) I, II and III only
(C) II, III and IV only
(D) III and IV only

4. If $5^x = 20$, then x =

(A) $\log\left(\frac{5}{20}\right)$

(B) $\log\left(\frac{20}{5}\right)$

(C) $\frac{\log 5}{\log 20}$

(D) $\frac{\log 20}{\log 5}$

5. $\log_c\left(\frac{a^2}{b}\right)$ can be written as

(A) $2\log_c a - \log_c b$

(B) $2\log_c\left(\frac{a}{b}\right)$

(C) $2\log_c\left(\frac{b}{a}\right)$

(D) $2\log_c b^a$

GO ON TO THE NEXT PAGE

6. $\frac{d}{dx}\left(e^{3x^2+2x+1}\right)$ is

(A) $(6x + 2)\, e^{6x-2}$
(B) $(6x + 2)\, e^{3x^2+2x+1}$
(C) $(3x^2 + 2x + 1)\, e^{6x-2}$
(D) $(3x^2 + 2x + 1)\, e^{3x^2+2x+1}$

7. A curve is defined parametrically by the equations $x = t^2$, $y = t(1 - t^2)$. The gradient of the curve, in terms of t, is

(A) $\frac{2t}{1-3t^2}$
(B) $\frac{1-3t^2}{2t}$
(C) $2t(1 - 2t)$
(D) $2t(1 + 2t)$

8. For $x^3y - 3 = -6x$, $\frac{dy}{dx}$ at the point where $x = 1$ and $y = -3$ is equal to

(A) -15
(B) $\frac{2}{3}$
(C) 3
(D) 11

9. Given $y = \ln(2x + 3)^3$, then $\frac{dy}{dx}$ is

(A) $\frac{2x}{2x+3}$
(B) $\frac{2}{2x+3}$
(C) $\frac{6x}{2x+3}$
(D) $\frac{6}{2x+3}$

10. If the function $f(x)$ is defined by $f(x) = \cos x$ then $f''(x)$ is

(A) $-\cos x$
(B) $-\sin x$
(C) $\cos x$
(D) $\sin x$

11. The partial fractions expression for $\frac{5}{(x+2)(x-3)}$ may be written as

(A) $\frac{1}{x+2}+\frac{1}{(x-3)}$
(B) $\frac{-1}{x+2}+\frac{1}{x-3}$
(C) $\frac{1}{x+2}+\frac{-1}{x-3}$
(D) $\frac{-1}{x+2}+\frac{-1}{x-3}$

12. Which of the following functions, when integrated w.r.t. x, gives the result $x - \ln x^2 + K$?

(A) $\frac{1}{1-x^2}$
(B) $\frac{1-2x}{x^2}$
(C) $\frac{x-2}{x}$
(D) $1-\frac{2}{x^2}$

GO ON TO THE NEXT PAGE

13. $\int \cos^2 x \, dx$ is equal to

(A) $\frac{1}{2} \sin^2 x + c$

(B) $\frac{1}{3} \cos^3 x + c$

(C) $\frac{1}{4} \sin 2x + c$

(D) $\frac{1}{4} \sin 2x + \frac{1}{2} x + c$

14. $\int \frac{x}{x^2+3} \, dx$ is equal to

(A) $\frac{1}{2} \ln (x^2 + 3) + c$
(B) $2 \ln (x^2 + 3) + c$
(C) $2x \ln (x^2 + 3) + c$
(D) $(x^2 + 3) \ln x + c$

15. $\int xe^{2x} dx$ may be expressed as

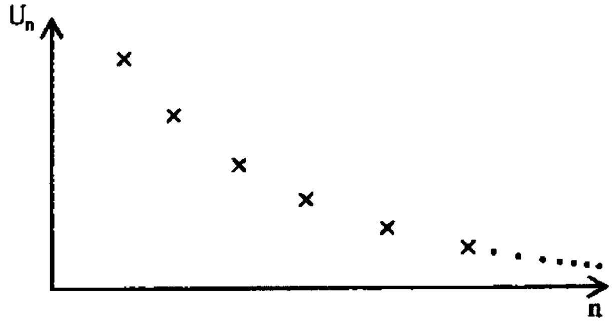
(A) $2xe^{2x} + e^{2x} + c$

(B) $2xe^{2x} - 4e^{2x} + c$

(C) $\frac{1}{2} xe^{2x} - \frac{1}{4} e^{2x} + c$

(D) $\frac{1}{2} xe^{2x} + \frac{1}{2} x^2 e^{2x} + c$

Item 16 refers to the diagram below.



16. The term that best describes the behaviour of the sequence $\{U_n\}$ shown above is

(A) periodic
(B) finite
(C) divergent
(D) convergent

Items **17 - 19** refer to S as defined below.

$S = 2^0 - 2^1 + 2^2 - 2^3 + 2^4$

17. S is best described as a

(A) finite series
(B) infinite series
(C) finite sequence
(D) infinite sequence

18. The general term in S is best defined by

(A) $(-1)^r \, 2^r$
(B) $(-1)^r \, (-2)^r$
(C) $(-1)^{r-1} \, (2)^r$
(D) $(-1)^{r-1} \, (-2)^{r-1}$

19. S may be written as

(A) $\sum_{r=0}^{r} (-1)^r (2^r)$

(B) $\sum_{r=1}^{r} (-1)^r (2)^{r-1}$

(C) $\sum_{r=0}^{4} (-1)^r (2)^r$

(D) $\sum_{r=1}^{4} (-1)^{r-1} (-2)^{r-1}$

Items **20 - 22** refer to E as defined below.

$$E = \sum_{n=19}^{30} \frac{3^n}{n}$$

20. The number of terms in the expansion of E is

(A) 10
(B) 11
(C) 12
(D) 13

21. The r^{th} term of E is 3^{24}. The value of r is

(A) 6
(B) 8
(C) 9
(D) 10

22. Which of the series below has its n^{th} term equal to $\frac{a}{2^n}$?

(A) $\sum_{r=1}^{n} \frac{a}{2^{-r}}$

(B) $\sum_{r=1}^{n} \frac{a}{2^r}$

(C) $\sum_{r=1}^{n} \frac{a}{2^{r-1}}$

(D) $\sum_{r=0}^{n} \frac{a}{2^{r-1}}$

23. $\sum_{r=1}^{20}(10-2r)=$

(A) -1410
(B) -220
(C) -10
(D) 220

24. The first term of an AP is 'a' and its common difference is -1. The sum of the first 10 terms is equal to

(A) 5(2a - 9)

(B) 5(2a + 9)

(C) 10(2a + 11)

(D) 10(2a - 11)

25. The second and fifth terms of a convergent geometric series with first term $\frac{81}{2}$ are 27 and 8 respectively. The sum to infinity of this series is

(A) $\frac{2}{3}$

(B) $\frac{243}{2}$

(C) $\frac{27}{2}$

(D) $\frac{81}{2}$

26. The sum to infinity of the geometric series $a + a^2 + a^3 + \ldots$ is $4a$ $(a \neq 0)$. The common ratio is

(A) $\frac{-3}{4}$

(B) $\frac{3}{4}$

(C) $\frac{4}{3}$

(D) $\frac{5}{4}$

27. ${}^nC_{r-1} =$

(A) $\frac{n!}{(n-r-1)!}$

(B) $\frac{n!}{(n-r+1)!}$

(C) $\frac{n!}{(n-r+1)!\,(r-1)!}$

(D) $\frac{n!}{[n-(r-1)]!\,r!}$

28. The 3rd term in the expansion of $(2 - \frac{x}{2})^6$ in ascending powers of x is

(A) $-60x^2$
(B) $-20x^3$
(C) $60x^2$
(D) $20x^3$

GO ON TO THE NEXT PAGE

29. The coefficient of x^2 in the expansion of $(2 - 3x)^5$ is

(A) -720
(B) -240
(C) 240
(D) 720

30. The function $f(x) = x^3 - 3x - 3$ has a root in the closed interval

(A) [-10, -8]
(B) [-2, 0]
(C) [2, 3]
(D) [5, 6]

31. The number of ways in which a committee of four men and six women can be seated in a row if they can sit in any position is

(A) 2!
(B) 4!
(C) 6!
(D) 10!

32. The number of ways in which 3 boys and 2 girls can sit so that no two persons of the same sex sit next to each other is

(A) 3 x 2
(B) 3! + 2!
(C) 3! x 2!
(D) 5!

33. A team of eleven players is to be chosen from a squad of 16 players. Given that 2 players must be chosen, the number of ways in which the team can be chosen is

(A) ${}^{14}C_{11}$
(B) ${}^{14}C_{9}$
(C) ${}^{16}C_{11}$
(D) ${}^{16}C_{9}$

34. In how many ways can two persons be selected from a group of 10 persons?

(A) 20
(B) 45
(C) 90
(D) 100

35. What is the probability that an integer chosen at random from 1, 2, 3, 4, 5, 6, 7, 9, 11, 15 is prime?

(A) $\frac{3}{10}$

(B) $\frac{4}{10}$

(C) $\frac{5}{10}$

(D) $\frac{6}{10}$

36. The letters P, Q, R, S, T and U are arranged randomly in a line. What is the probability that P and Q are next to each other?

(A) $\frac{2 \times 5!}{6!}$

(B) $\frac{2 \times 6!}{5!}$

(C) $\frac{5 \times 2!}{6!}$

(D) $\frac{5 \times 6!}{2!}$

GO ON TO THE NEXT PAGE

37. What is the probability that an integer chosen at random from 1, 2, 3, 4, 5, 6, 7, 9 11, 15 is divisible by 3?

(A) $\frac{1}{10}$

(B) $\frac{2}{10}$

(C) $\frac{3}{10}$

(D) $\frac{4}{10}$

38. A and B are two independent events. Given P (A) = 0.3 and P (B) = 0.4, which of the following are true?

I. $P(A \cup B) = 0.7$
II. $P(A \cap B) = 0.12$
III. $P(A \mid B) = 0.3$
IV. $P(A \mid \overline{B}) = 0.4$

(A) I and II only
(B) II and III only
(C) II and IV only
(D) III and IV only

39. A fair die is tossed twice. What is the probability that at least one toss results in a 5?

(A) $\frac{2}{36}$

(B) $\frac{10}{36}$

(C) $\frac{11}{36}$

(D) $\frac{25}{36}$

Items 40 refers to the matrices P and Q below.

$$P = \begin{bmatrix} a & b & c \end{bmatrix}, Q = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

40. The product PQ is

(A) [ax + by + cz]

(B) $\begin{bmatrix} ax \\ by \\ cz \end{bmatrix}$

(C) [ax by cz]

(D) not possible

Items 41 - 42 refer to the matrix $\begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix}$

41. The cofactor of the circled element, -2, is

(A) -2
(B) -1
(C) 0
(D) 2

42. The determinant of the given matrix is

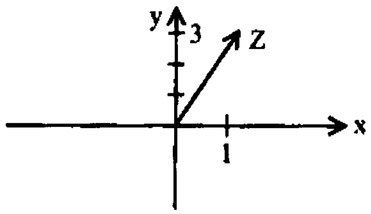
(A) -5
(B) -3
(C) 3
(D) 5

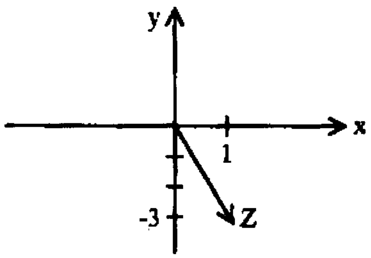
43. If z is the complex number 2 - i, then z^2 equals

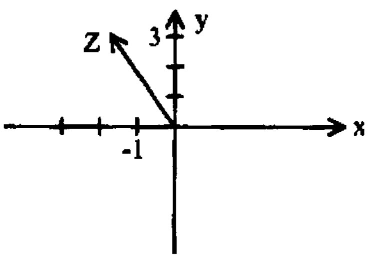
(A) 3
(B) 4
(C) 3 - 4i
(D) 4 - 3i

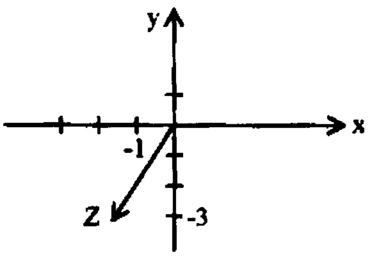
GO ON TO THE NEXT PAGE

44. Which Agrand diagram best represents the complex number $z = 1 - i\sqrt{8}$?

(A) 

(B) 

(C) 

(D) 

45. Determine Im $\left(\frac{1}{z}\right)$, where $z = \frac{3 - i}{1 + i}$

(A) $-\frac{2}{5}$

(B) $-\frac{1}{5}$

(C) $\frac{1}{5}$

(D) $\frac{2}{5}$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

FORM TP 02234020/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 02

2 hours 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 6 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) (i) Using the fact that $e^{-x} = \frac{1}{e^x}$ or otherwise, show that,

$$\frac{d}{dx}(e^{-x}) = -e^{-x}.$$ **[2 marks]**

(ii) Hence, evaluate $\int x^2 e^{-x}\, dx$. **[4 marks]**

(b) (i) a) Find $\frac{dy}{dx}$ when $y = \tan^{-1}(3x)$. **[4 marks]**

b) Hence, find $\int \frac{(x+2)}{1+9x^2}\, dx$. **[4 marks]**

(ii) Show that if $y = \frac{\ln(5x)}{x^2}$ then $\frac{dy}{dx} = \frac{1-\ln(25x^2)}{x^3}$. **[6 marks]**

(c) Solve the first order differential equation

$$y \tan x \frac{dy}{dx} = (4 + y^2)\sec^2 x.$$ **[5 marks]**

Total 25 marks

2 (a) In 1950, the world population was 2.5 billion and it grew to 5 billion in 1987. The world's population grows exponentially so that at time t years the population is $N = 2.5\, e^{kt}$ where t =0 corresponds to the year 1950 and N is measured in billions of people.

Find

(i) the exact value of k **[3 marks]**

(ii) the exact value of N in 2003 **[2 marks]**

(iii) the year in which N =10. **[5 marks]**

(b) Given that $y = u \cos 3x + v \sin 3x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} 3y = -30 \sin 3x,$$

Find

(i) the values of the constants u and v **[10 marks]**

(ii) the general solution of the differential equation. **[5 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION B (MODULE 2)

Answer BOTH questions.

3. (a) (i) Find constants A and B such that

$$\frac{1}{(2r-1)(2r+1)} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}.$$

[5 marks]

(ii) Hence, find the value of S where

$$S = \sum_{r=1}^{n} \frac{1}{2r-1)(2r+1)}.$$

[5 marks]

(iii) Deduce the sum to infinity of S. **[3 marks]**

(b) (i) Find the n^{th} term of the series 1(2) + 2(5) + 3(8) +… **[2 marks]**

(ii) Prove, by Mathematical Induction, that the sum to n terms of the series in (b)(i) above is $n^2(n + 1)$. **[10 marks]**

Total 25 marks

4. (a) Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + ...$

(i) show that the series is geometric **[4 marks]**

(ii) find the sum of the series to n terms. **[4 marks]**

(b) Use Maclaurin's Theorem to find the <u>**first**</u> three non-zero terms in the power series expansion of cos $2x$. **[7 marks]**

(c) (i) Expand up to and including the term in x^3

$$\sqrt{\left(\frac{1+x}{1-x}\right)},$$

stating the values of x for which the expansion is valid. **[6 marks]**

(ii) By taking $x = 0.02$ find an approximation for $\sqrt{51}$, correct to 5 decimal places. **[4 marks]**

Total 25 marks

SECTION C (MODULE 3)

Answer BOTH questions.

5. (a) Two cards are drawn without replacement from ten cards which are numbered 1 to 10. Find the probability that

(i) the numbers on **BOTH** cards are even **[4 marks]**

(ii) the number on one card is odd and the number on the other card is even. **[4 marks]**

(b) A journalist reporting on criminal cases classified 150 criminal cases by the age (in years) of the criminal and by the type of crime committed, violent or non-violent. The information is presented in the table below.

Type of Crime	**Age (in years)**		
	Less than 20	**20 to 39**	**40 or older**
Violent	27	41	14
Non-violent	12	34	22

What is the probability that a case randomly selected by the journalist

(i) a) is a violent crime? **[2 marks]**

b) was committed by someone LESS than 40 years old? **[4 marks]**

c) is a violent crime OR was committed by a person LESS than 20 years old? **[5 marks]**

d) is a violent crime that was committed by a person LESS than 20 years old? **[2 marks]**

(ii) Two criminal cases are randomly selected for review by a judge. What is the probability that BOTH cases are violent crimes? **[4 marks]**

Total 25 marks

6. (a) Solve the following equation using determinants:

$$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

[10 marks]

(b) Solve the following set of equations:

$$\begin{aligned} x_1 - 4x_2 - 2x_3 &= 21 \\ 2x_1 + x_2 + 2x_3 &= 3 \\ 3x_1 + 2x_2 - x_3 &= -2 \end{aligned}$$

[10 marks]

(c) (i) Express the complex number $\dfrac{4-2i}{1-3i}$ in the form of $a + bi$ where a and b are real numbers. **[4 marks]**

(ii) Show that the argument of the complex number in (c)(i) above is $\dfrac{\pi}{4}$.

[1 mark]

Total 25 marks

END OF TEST

FORM TP 02234032/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 03B

$1\frac{1}{2}$ ***hours***

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

The maximum mark for each question is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL THREE** questions.

3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

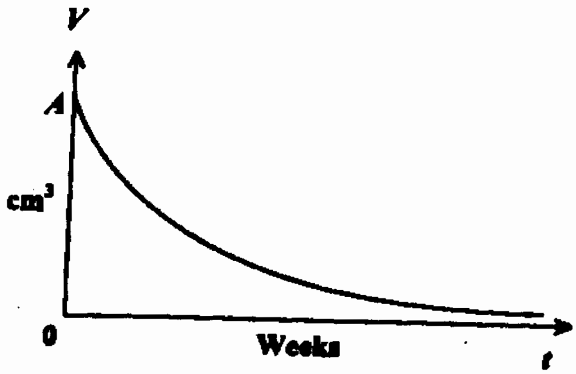
Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer this question.

1. The diagram below, **not drawn to scale,** shows the variation in the volume, V cm^3, of an air freshener block with time, t weeks.



The variation can be written as

$$\frac{dV}{dt} = -kV.$$

(a) Describe clearly in words the variation shown in the diagram above. **[2 marks]**

(b) Show that $V = Ae^{-kt}$, where A is a constant. **[5 marks]**

(c) Initially, an air freshener block has a volume of 64 cm^3. It loses half its volume after 6 weeks. Show that $V = 64e^{\left(\ln\frac{1}{2}\right)\frac{t}{6}}$. **[8 marks]**

(d) The air freshener block becomes ineffective when its volume reaches 6 cm^3. Calculate the time, to the NEAREST week, at which the block should be replaced. **[5 marks]**

Total 20 marks

SECTION B (MODULE 2)

Answer this question.

2. (a) The sum to infinity of a GP is 10 times the first term. Find the common ratio of the GP. **[3 marks]**

(b) A ball is projected vertically upwards to a height of 81 cm above a horizontal floor. It drops to the floor and bounces until it comes to a stop. After each bounce on the floor, the ball rises vertically to a height that is $\frac{2}{3}$ of the distance it dropped.

(i) Find

a) h_1, the distance the ball travels before the first bounce
b) h_2, the distance the ball travels before the second bounce
c) h_3, the distance the ball travels before the third bounce.

[3 marks]

(ii) Given that h_n, $n > 2$, is the distance the ball has travelled between the nth bounce and the **previous** bounce, express

a) h_n in terms of h_{n-1}
b) h_{n-1} in terms of h_{n-2}. **[2 marks]**

(iii) Hence, show that

$$h_n = \left(\frac{2}{3}\right)^{n-1} h_1$$ **[3 marks]**

(iv) Show that the TOTAL distance that the ball has travelled just before the nth bounce is

$$486\left(1-\left(\frac{2}{3}\right)^n\right) \text{ cm.}$$ **[6 marks]**

(v) Deduce that the TOTAL distance travelled by the ball when it stops bouncing is approximately 486 cm. **[3 marks]**

Total 20 marks

SECTION C (MODULE 3)

Answer this question.

3. (a) (i) A singer is scheduled to sing 6 particular songs without repetition at a cultural show. In how many ways can this singer's schedule be arranged? **[2 marks]**

(ii) If the singer has altogether 13 suitable songs, in how many ways can a schedule of 6 songs be prepared? **[3 marks]**

(b) A biology examination includes 4 True or False questions. The probability of a student guessing the correct answer to any question is $\frac{1}{2}$. Use the probability model

$$P(r) = \frac{n!}{r!(n-r)!}p^r q^{n-r}$$

where n is the number of questions
r is the number of observed successes
p is the probability of guessing correctly
q is the probability of guessing incorrectly

to answer the questions below.

What is the probability of a student guessing the correct answer to

(i) At LEAST ONE of the four questions correctly? **[5 marks]**

(ii) EXACTLY ONE of the four questions correctly? **[2 marks]**

(c) The transformation in three-dimensional space of a point, P, with coordinates (x, y, z) is represented below.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$$

By row-reducing the augmented matrix, find the coordinates of P. **[8 marks]**

Total 20 marks

02234010/CAPE/K - 2007

CARIBBEAN EXAMINATIONS COUNCIL
HEADQUARTERS

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 2

PAPER 01

KEY

CARIBBEAN EXAMINATIONS COUNCIL

Pure Mathematics Unit 2

Item	Key	Item	Key
1	D	24	A
2	A	25	B
3	C	26	B
4	D	27	C
5	A	28	C
6	B	29	D
7	B	30	C
8	C	31	D
9	D	32	C
10	A	33	B
11	B	34	B
12	C	35	D
13	D	36	A
14	A	37	D
15	C	38	B
16	D	39	C
17	A	40	A
18	A	41	A
19	C	42	C
20	C	43	C
21	C	44	B
22	B	45	D
23	B		

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

PAPER 02

SOLUTIONS
&
MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a) (i) $\frac{d}{dx}\left(e^{-x}\right) = \frac{d}{dx}\left(\frac{1}{e^x}\right)$

$= \frac{e^x(0)-(1)\left(e^x\right)}{\left(e^x\right)^2}$ (1 mark)

$= \frac{-e^x}{\left(e^x\right)^2}$

$= \frac{-1}{e^x}$

$= -e^{-x}$ (1 mark)

[2 marks]

(ii) $\int x^2 e^{-x}\,dx$; let $u = x^2$ so that $du = 2x\,dx$, and

let $dv = e^{-x}\,dx$ so that $v = -e^{-x}$, then (1 mark)

$\int x^2 e^{-x} dx = -x^2 e^{-x} + 2\int e^{-x} x\,dx$ (1 mark)

$= -x^2 e^{-x} + 2\left[-xe^{-x} + \int e^{-x} dx\right]$ (1 mark)

$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + c$ (1 mark)

i.e. $= -e^{-x}\left[x^2 + 2x + 2\right] + c$ [4 marks]

(b) (i) a) $y = \tan^{-1}(3x) \Rightarrow \tan y = 3x$ (1 mark)

$\Rightarrow \sec^2 y\,\frac{dy}{dx} = 3$

$\Rightarrow \frac{dy}{dx} = \frac{3}{\sec^2 y}$ (1 mark)

$\Rightarrow \frac{dy}{dx} = \frac{3}{1+\tan^2 y}$ (1 mark)

$\Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2}$ (1 mark)

[4 marks]

b) $\int \frac{x+2}{1+9x^2}\,dx = \int \frac{2}{1+9x^2}\,dx + \int \frac{x}{1+9x^2}\,dx$ (1 mark)

$= \frac{2}{3}\tan^{-1}(3x) + \frac{1}{18}\ln(1+9x^2) + k$ (3 marks)

[4 marks]

(ii) $y = \frac{\ln(5x)}{x^2}$, Using quotient rule :

$\Rightarrow \frac{dy}{dx} = \frac{x^2\frac{1}{x} - (2x)\ln(5x)}{x^4}$ (3 marks)

$= \frac{1-2\ln(5x)}{x^3}$ (1 mark)

$= \frac{1-\ln(5x)^2}{x^3}$ (1 mark)

$= \frac{1-\ln(25x^2)}{x^3}$ (1 mark)

[6 marks]

(c) $y\tan x\,\frac{dy}{dx} = (4+y^2)\sec^2 x$

$\frac{y}{4+y^2}\frac{dy}{dx} = \frac{\sec^2 x}{\tan x}$ (1 mark)

$\frac{y\,dy}{4+y^2} = \frac{\sec^2 x\,dx}{\tan x}$ (1 mark)

$\int \frac{y\,dy}{4+y^2} = \int \frac{\sec^2 x\,dx}{\tan x}$ (1 mark)

$\frac{1}{2}\ln(4+y^2) = \ln\tan x + c$ (2 marks)

[5 marks]

Total 25 marks

Specific Objectives: (b) 1, 5, 6; (c) 5, 6, 7, 8, 9,10

Question 2

(a) $N = 2.5\ e^{kt}$ (given)

(i) In 1987, t = 37 and N = 5.0 (1 mark)

$\Rightarrow 5.0 = 2.5e^{k}$ (1 mark)

$\Rightarrow e^{37k} = 2 \Rightarrow k = \frac{1}{37}\ln 2$ (1 mark)

[3 marks]

(ii) In 2003, $t = 53 \Rightarrow N = 2.5 \times e^{\frac{53}{57}}\ln 2 = 2.5(\ln 2)^{\frac{53}{37}}$ (2 marks)

[2 marks]

(iii) $N = 10 \Rightarrow \quad 10 = 2.5^{kt} \Rightarrow e^{kt} \Rightarrow e^{kt} = 4$ (2 marks)

$\Rightarrow \quad kt = \ln 4 \quad = 2\ln 2$ (1 marks)

$\Rightarrow \quad t = \frac{2}{k}\ln 2 = 2 \times 37 = 74$ (1 mark)

i.e. N = 10 in the year 2024 (1 mark)

[5 marks]

(b) (i) $y = u\cos 3x + v\sin 3x$

$\Rightarrow \frac{dy}{dx} = -3u\sin 3x + 3v\cos 3x$ (2 marks)

$\Rightarrow \frac{d^2y}{dx^2} = -9u\cos 3x - 9v\sin 3x$ (2 marks)

so, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = -30\sin 3x$

$\Rightarrow -(6v - 12u)\sin 3x + (-6u + 12v)\cos 3x = -30\sin 3x$ (2 marks)

$\Rightarrow \quad 2u + v = 5$ and $u = 2v$ (2 marks)

$\Rightarrow u = 2$ and $v = 1$. (2 marks)

[10 marks]

(ii) the auxiliary equation of the different equation is

$$k^2 + 4k + 3 = 0$$ (1 mark)

$\Rightarrow \quad (k+3)(k+1) = 0$ (1 mark)

$\Rightarrow \quad k = -3 \text{ or } -1$

$\Rightarrow$ the complementary function is

$y = Ae^{-3x} + Be^{-x}$; A, B constants (2 marks)

General solution is $y = Ae^{-3x} + Be^{-x} + \sin 3x + 2\cos 3x$. (1 mark)

[5 marks]

Total 25 marks

Specific Objectives: (a) 7, 8, 9, 10; (b) 1, 4, 5, 7

SECTION B

(MODULE 2)

Question 3

(a) (i)

$$\frac{1}{(2r-1)(2r+1)} = \frac{A}{2r-1} + \frac{B}{2r+1}$$

$\Rightarrow \quad 1 = A(2r+1) + B(2r-1)$ (1 mark)

$\Rightarrow \quad 0 = 2A + 2B$ and $A - B = 1$ (2 marks)

$\Rightarrow \quad A = \frac{1}{2}$ and $B = -\frac{1}{2}$ (2 marks)

[5 marks]

(ii) $S = \sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \sum_{r=1}^{n} \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$ (1 mark)

$= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \ldots + \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$

(3 marks)

$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$ (1 mark)

[5 marks]

(iii) As $n \to \infty$, $\frac{1}{2n+1} \to 0$ (2 marks)

Hence $S_\infty = \frac{1}{2}$ (1 mark)

[3 marks]

(b) (i) $S = 1(2) + 2(5) + 3(8) + \ldots$

In each term, 1st factor is in the natural sequence and the second factor differs by 3 (1 mark)

$\Rightarrow$ the nth term is $n\,(3n - 1)$ (1 mark)

[2 marks]

(ii) $S_n = \sum_{r=1}^{n} r(3r-1)$

for n = 1, $S_1 = \sum_{r=1}^{1} r(3r-1) = 1 x 2 = 2$

and $1^2\,(1+1) = 1 x 2 = 2$ (1 mark)

hence $S_n = n^2\,(n+1)$ is true for n = 1 (1 mark)

Assume $S_n = n^2\,(n+1)$ for $n = k \in N$

that is, $S_k = k^2\,(k+1)$ (1 mark)

Then, $S_{k+1} = \sum_{r=1}^{k+1} r\,(3r-1) = S_k + (k+1)(3k+2)$ (1 mark)

$= k^2(k+1) + (k+1)(3k+2)$ (1 mark)

$= (k+1)\left[k^2 + 3k + 2\right]$ (1 mark)

$\Rightarrow$ $S_{k+1} = (k+1)\left[(k+1)(k+2)\right]$ (1 mark)

$= (k+1)^2\,\left[(k+1)+1\right]$ (1 mark)

$\Rightarrow$ true for n = k+1 whenever it is assumed true for n = k, (1 mark)

$\Rightarrow$ true for all $n \in N$

$\Rightarrow$ $S_n = n^2(n+1) \quad \forall n \in N$ (1 mark)

[10 marks]

Total 25 marks

Specific Objectives: (b) 1, 3, 5, 6, 10

Question 4

(a) (i) Let $S \equiv \frac{1}{2}+\frac{1}{2^4}+\frac{1}{2^7}+\frac{1}{2^{10}}+\dots$

S is a geometric series

$\Leftrightarrow \quad \frac{T_2}{T_1}=\frac{T_n}{T_{n-1}}=r$

i.e. S has common ratio r (1 mark)

$$\frac{\frac{1}{2^4}}{\frac{1}{2}} = \frac{\frac{1}{2^7}}{\frac{1}{2^4}}$$ (1 mark)

$$= \frac{1}{2^3}$$ (1 mark)

$\therefore$ S is geometric with common ratio $r=\frac{1}{2^3}$ (1 mark)

[4 marks]

(ii) $$S_n = \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^{3n}\right]}{1-\left(\frac{1}{2}\right)^3}$$ (1 mark)

$$= \frac{\frac{1}{2}\left[1-\frac{1}{2^{3n}}\right]}{1-\frac{1}{8}}$$ (1 mark)

$$= \frac{1}{2}\times\frac{8}{7}\left[1-\frac{1}{2^{3n}}\right]$$ (1 mark)

$$= \frac{4}{7}\left[1-\frac{1}{2^{3n}}\right]$$ (1 mark)

[4 marks]

(b) (i) $f(x)=\cos 2x \quad \Rightarrow \quad f^{i}(x)=-2\sin 2x$ (1 mark)

$\Rightarrow \quad f^{ii}(x)=-4\cos 2x$ (1 mark)

$\Rightarrow \quad f^{iii}(x)=8\sin 2x$ (1 mark)

$\Rightarrow \quad f^{iv}(x)=16\cos 2x$ (1 mark)

so, $f(0)=1,\ f^{i}(0)=0,\ f^{ii}(x)=-4,\ f^{iii}(0)=0,\ f^{iv}(0)=16$ (1 mark)

Hence, by Maclaurin's Theorem,

$$\cos 2x = 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} -$$ (1 mark)

$$= 1 - 2x^2 + \frac{2}{3}x^4$$ (1 mark)

[7 marks]

(c) (i) $\sqrt{\left(\dfrac{1+x}{1-x}\right)}$

$$= (1+x)^{\frac{1}{2}}\ (1-x)^{-\frac{1}{2}}$$ (1 mark)

$$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3\ldots\right)\left(1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{5}{16}x^3\ldots\right)$$ (2 marks)

$$= 1+x+\frac{1}{2}x^2+\frac{1}{2}x^3$$ (2 marks)

for $-1<x<1$ (1 mark)

[6 marks]

(ii) $\sqrt{\dfrac{1.02}{0.98}} = \sqrt{\dfrac{102}{98}} = \dfrac{1}{7}\sqrt{51}$ (1 mark)

$\sqrt{51} \ = 7\sqrt{\dfrac{1+x}{1-x}}$ where $x = 0.02$ (1 mark)

$\Rightarrow \sqrt{51} \ = 7\left\{1+0.02+\dfrac{1}{2}(0.02)^2+\dfrac{1}{2}(0.02)^3\right\}$ (1 mark)

$= 7.14141$ (5 d.p.) (1 mark)

[4 marks]

Specific Objectives: (b) 5, 9, 11; (c)3, 4

Total 25 marks

SECTION C

(MODULE 3)

Question 5

(a) (i) P (First card drawn has even number) $= \frac{5}{10} = \frac{1}{2}$ (1 mark)

P (Second card drawn has even number) $= \frac{4}{9}$ (2 marks)

$\therefore$ P (Both cards have even numbers) $= \left(\frac{1}{2}\right)\left(\frac{4}{9}\right)$

$= \frac{2}{9}$ (1 mark)

[4 marks]

(ii) P (Both cards have odd numbers) $= \frac{2}{9}$ (1 mark)

P (One card has odd and the other has even i.e. both cards do not have odd or do not have even numbers) $= 1 - 2\left(\frac{2}{9}\right)$ (2 marks)

$= \frac{5}{9}$ (1 mark)

[4 marks]

(b) (i) a) $\frac{82}{150} = 0.547$ [2 marks]

b) $\frac{39}{150} + \frac{75}{150} = 0.76$ [4 marks]

c) $\frac{82}{150} + \frac{39}{150} - \frac{27}{150} = 0.267$ [5 marks]

d) $\frac{27}{82} = 0.329$ [2 marks]

(ii) $\frac{82}{150} \times \frac{81}{149} = 0.297$ [4 marks]

Total 25 marks

Specific Objectives: (a) , 5, 6, 7, 8, 9, 10, 11, 12, 13

Question 6

(a) $$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

$$5\,(2x-2) - x\,(x^2+2x+3) + 3\,(2x+4+6) = 0$$ (5 marks)

$$x^3 + 2x^2 - 13x - 20 = 0$$ (1 mark)

Subs $x = -4$, $(-4)^3 + 2\,(-4)^2 - 13(-4) - 20 = 0$

$$(x+4)\left(x^2 - 2x - 5\right) = 0$$ (2 marks)

$$x = -4$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = 1 \pm \sqrt{6}$$ (2 marks)

[10 marks]

(b) Writing the equations in matrix form.

$$\begin{pmatrix} 1 & -4 & -2 \\ 2 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 21 \\ 3 \\ -2 \end{pmatrix}$$

The augmented matrix is

$$\left(\begin{array}{ccc|c} 1 & -4 & -2 & 21 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & -1 & -2 \end{array}\right)$$

(1 mark)

Eliminate x_1 from Row 2 and Row 3
Subtract 2 x Row_1 from Row_2 and 3 x Row_1 from Row_3 to give

$$\left(\begin{array}{ccc|c} 1 & -4 & -2 & 21 \\ 0 & 9 & 6 & -39 \\ 0 & 14 & 5 & -65 \end{array}\right)$$ (2 marks)

Row 3 $-\ \frac{14}{9}$ Row 2 gives

$$\left(\begin{array}{ccc|c} 1 & -4 & -2 & 21 \\ 0 & 9 & 6 & -39 \\ 0 & 0 & \frac{-13}{3} & \frac{-13}{3} \end{array}\right)$$

(2 marks)

Referencing the matrix gives

$$\begin{pmatrix} 1 & -4 & -2 \\ 0 & 9 & 6 \\ 0 & 0 & \frac{-13}{3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 21 \\ -39 \\ \frac{-13}{3} \end{pmatrix}$$

$x_1 = 3$, $x_2 = -5$, $x_3 = 1$
by "back substitution".

$$\left[\begin{array}{ll} x_3 & -\text{ 1 mark} \\ x_2,\ x_1 & -\text{ 2 marks each} \end{array}\right]$$

(5 marks)

[10 marks]

(c) (i) $\frac{4-2i}{1-3i} = \frac{(4-2i)(1+3i)}{(1-3i)(1+3i)}$ (1 mark)

$= \frac{4+12i-2i-6i^2}{1-9i^2}$ (1 mark)

$= \frac{4+10i+6}{1+9}$ (1 mark)

$= \frac{10+10i}{10}$

$= 1+i$ (1 mark)

[4 marks]

(ii) arg is $\tan^{-1}(\frac{1}{1}) = \frac{\pi}{4}$ (1 mark)

[1 mark]

Total 25 marks

Specific Objectives: (b) 1, 2, 3, 4, 5, 6, 7, 8; (c) 4, 5, 6, 7

02234032/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 03B

SOLUTIONS
&
MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a) The volume of the air freshener decreases exponentially with time.

At $t = 0$, the volume is A cm^3. 1 mark
As $t \to \infty$, the volume $\to 0$. 1 mark
[2 marks]

(b) $\int \frac{1}{V}\, dV = \int -k\,dt$ 1 mark

$\ln V = -kt + c$ 2 marks

$V = e^{-kt+c}$ 1 mark
$= e^{-kt}\, e^{c}$
$= Ae^{-kt}$ where $A = e^{c}$. 1 mark
[5 marks]
OR

i.e. $V = Ae^{-kt}$, as required

OR

$\ln V = -kt + \ln A$ 1 mark
$\ln V - \ln A = -kt$ 1 mark
$\ln \frac{V}{A} = -kt$ 1 mark
$\frac{V}{A} = e^{-kt}$ 1 mark

$V = Ae^{-kt}$, as required 1 mark
[5 marks]

(c) $V = Ae^{-kt}$
At $t = 0$, $V = 64$:
$64 = A(1)$ 1 mark
$A = 64$
$\therefore V = 64e^{-kt}$ 1 mark

At $t = 6$, $V = 32$ 1 mark
$32 = 64e^{-6k}$

$\frac{1}{2} = e^{-6k}$ 1 mark

$\ln \frac{1}{2} = -6k$ 1 mark

$k = -\frac{1}{6} \ln \frac{1}{2}$ 1 mark

$\therefore \quad V = 64e^{-\left(-\frac{1}{6} \ln \frac{1}{2}\right) t}$ 1 mark

$V = 64e^{\left(\ln\frac{1}{2}\right)\left(\frac{t}{6}\right)}$ 1 mark

[8 marks]

(d) Now $V = 64e^{\left(\ln \frac{1}{2}\right)\left(\frac{t}{6}\right)}$

$\Rightarrow \quad V = 64\left(\frac{1}{2}\right)^{\frac{t}{6}}$ 1 mark

Block is ineffective for V = 6 1 mark

$\therefore \quad 6 = 64\left(\frac{1}{2}\right)^{\frac{t}{6}}$ 1 mark

$\frac{3}{32} = \left(\frac{1}{2}\right)^{\frac{t}{6}}$

$\ln \frac{3}{32} = \frac{t}{6} \ln \left(\frac{1}{2}\right)$

$t = \frac{6 \ln \frac{3}{32}}{\ln \frac{1}{2}}$ 1 mark

≈ 20.49

≈ 20

Ans 20 weeks 1 mark

[5 marks]

Total 20 marks

Specific Objectives: (a) 3, 7, 8, 9, 11; (c) 11

SECTION B

(MODULE 2)

Question 2

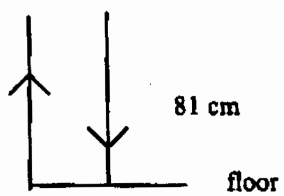
(a) $S_\infty = \dfrac{a}{1-r}$

So $\dfrac{a}{1-r} = 10a$ — 1 mark

$\Rightarrow 10(1-r) = 1$ — 1 mark

$\Rightarrow r = \frac{9}{10}$ — 1 mark

[3 marks]



(b) (i) a) $h_1 = 2 \text{ x } 81 \text{ cm} = 162 \text{ cm}$ — 1 mark

b) $h_2 = \dfrac{2}{3} \text{ x } 81 \text{ x } 2 \text{ cm} = \dfrac{2}{3} h_1$ — 1 mark

c) $h_3 = \dfrac{2}{3} \text{ x } \dfrac{2}{3} \text{ x } 81 \text{ x } 2 \text{ cm} = \dfrac{2}{3} h_2$ — 1 mark

[3 marks]

(ii) a) Hence

$h_n = \dfrac{2}{3} h_{n-1}$ — 1 mark

b) $h_{n-1} = \dfrac{2}{3} h_{n-2}$ — 1 mark

[2 marks]

(iii) So, $h_n = \dfrac{2}{3} h_{n-1}, n > 1$

$= \dfrac{2}{3} \text{ x } \dfrac{2}{3} h_{n-2}, n > 2$ — 1 mark

$= \underbrace{\dfrac{2}{3} \text{ x } \dfrac{2}{3} \text{ x} \ldots \text{x } \dfrac{2}{3}}_{n-1 \text{ times}} h_1$ — 1 mark

$= \left(\dfrac{2}{3}\right)^{n-1} h_1$ — 1 mark

[3 marks]

(iv) Distance $= h_1 + h_2 + \ldots + h_n$ 1 mark

$= h_1 + \frac{2}{3}h_1 + \left(\frac{2}{3}\right)^2 h_1 + \ldots + \left(\frac{2}{3}\right)^{n-1} h_1$ 1 mark

$= h_1\left(1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \ldots + \left(\frac{2}{3}\right)^{n-1}\right)$ 1 mark

$= h_1\left(\dfrac{1 - \left(\frac{2}{3}\right)^n}{1 - \frac{2}{3}}\right)$ 1 mark

$= 162\left(\dfrac{1 - \left(\frac{2}{3}\right)^n}{\frac{1}{3}}\right)$ 1 mark

$= 486\left(1 - \left(\frac{2}{3}\right)^n\right)$ cm 1 mark

[6 marks]

(v) As $n \to \infty$, $\left(\frac{2}{3}\right)^n \to 0$. 2 marks

Hence, when the ball stops bouncing the distance is approximately 486 cm. 1 mark

[3 marks]

Total 20 marks

Specific Objectives: (a)3, 4; (b) 5, 9, 10, 12; GO:6

SECTION C

(MODULE 3)

Question 3

(a) (i)

Number of ways of choosing 1^{st} song is 6
Number of ways of choosing 2^{nd} song is 5
Number of ways of choosing 3^{rd} song is 4
Number of ways of choosing 4^{th} song is 3
Number of ways of choosing 5^{th} song is 2
Number of ways of choosing 6^{th} song is 1

1 mark

Total number of ways is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$ 1 mark

[2 marks]

(ii)

Number of ways of choosing 1^{st} song is 13 1 mark
Number of ways of choosing 2^{nd} song is 12
Number of ways of choosing 3^{rd} song is 11
Number of ways of choosing 4^{th} song is 10
Number of ways of choosing 5^{th} song is 9
Number of ways of choosing 6^{th} song is 8 1 mark

Total number of ways is $13 \times 12 \times 11 \times 10 \times 9 \times 8 = {}^{13}P_7$ 1 mark

$= 1\ 235\ 520$

[3 marks]

(b) (i)

P (None correct) = P (r = 0) 1 mark

P (incorrect guess) = 1 – 0.5 1 mark

$$P(r=0) = \frac{4!}{(0!)\,(4-0)!}\,(0.50)^0\,(1-0.50)^{4-0}$$ 1 mark

$$= \frac{4 \times 3 \times 2 \times 1}{(1)(4 \times 3 \times 2 \times 1)}\,(1)(0.50)^4$$

$$= (0.50)^4$$

$$= 0.0625$$ 1 mark

$$P(r \geq 1) = 1 - 0.0625$$
$$= 0.938$$ 1 mark

[5 marks]

(ii) $$\mathrm{P}(r=1) = \frac{4!}{(0!)\,(4-1)!}\,(0.50)^1\,(1-0.50)^{4-1}$$ 1 mark

$$= \frac{4 \times 3 \times 2 \times 1}{(1)\,(3 \times 2 \times 1)}\,(0.50)\,(0.50)^3$$

$$= 4\,(0.50)^4 = 0.2500$$ 1 mark

[2 marks]

(c) Augmented matrix $= \left(\begin{array}{ccc|c} 1 & 4 & 1 & 7 \\ 1 & -1 & 2 & 9 \\ 2 & 1 & -1 & 1 \end{array}\right)$ 1 mark

$$r_3 - 2r_1 \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 1 & -1 & 2 & 9 \\ 0 & -1 & -3 & -13 \end{array}\right)$$ 1 mark

$$r_2 - r_1 \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 2 & 2 \\ 0 & -1 & -3 & -13 \end{array}\right)$$ 1 mark

$$2r_3 - r_2 \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 2 \\ 0 & 0 & -7 & -28 \end{array}\right)$$ 1 mark

Re-writing the matrix gives

$$\begin{aligned} x + y + z &= 7 \\ -2y + z &= 2 \\ -7z &= -28 \end{aligned}$$

Solving for x, y and z gives $z = 4, y = 1, x = 2$ 3 marks

$$\mathrm{P} = (2, 1, 4)$$ 1 mark

[8 marks]

Total 20 marks

Specific Objectives: (a) 2, 4, 7, 9, 10 (b) 3, 4, 5; GO: 5