



CARIBBEAN EXAMINATIONS COUNCIL  
**ADVANCED PROFICIENCY EXAMINATION**

**PURE MATHEMATICS**

**UNIT 2 – PAPER 03/B**

**ANALYSIS, MATRICES AND COMPLEX NUMBERS**

*1 ½ hours*

**02 JUNE 2010 (a.m.)**

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 4 printed pages.

**INSTRUCTIONS TO CANDIDATES**

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

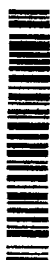
**Examination Materials Permitted**

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2009**

Mathematical instruments

Silent, non-programmable, electronic calculator



**SECTION A (Module 1)**

**Answer this questions.**

1. (a) Express in partial fractions

$$\frac{1 - x^2}{x(x^2 + 1)}.$$

**[7 marks]**

- (b) The rate of change of a population of bugs is modelled by the differential equation  $\frac{dy}{dt} - ky = 0$ , where  $y$  is the size of the population at time,  $t$ , given in days, and  $k$  is the constant. Initially, the population is  $y_0$  and it doubles in size in 3 days.

- (i) Show that

a)  $y = y_0 e^{kt}$

**[7 marks]**

b)  $k = \frac{1}{3} \ln 2.$

**[3 marks]**

- (ii) Find the proportional increase in population at the end of the second day.

**[3 marks]**

**Total 20 marks**

**SECTION B (Module 2)**

**Answer this questions.**

2. (a) The sum to infinity of a convergent geometric series is equal to six times the first term. Find the common ratio of the series. **[5 marks]**

- (b) Find the sum to infinity of the series  $\sum_{r=1}^{\infty} a_r$  whose  $r$  th term  $a_r$  is

$$\frac{2r+1}{r!}.$$

**[8 marks]**

- (c) A truck bought for \$15 000 depreciates at the rate of  $12\frac{1}{2}\%$  each year. Calculate the value of the truck

- (i) after 1 year **[2 marks]**
- (ii) after  $t$  years **[2 marks]**
- (iii) when its value FIRST falls below \$5 000. **[3 marks]**

**Total 20 marks**

**GO ON TO THE NEXT PAGE**

**SECTION C (Module 3)**

**Answer this questions.**

3. (a) Find the number of integers between 300 and 1 000 which can be formed by using the digits 1, 3, 5, 7 and 9
- (i) if NO digit can be repeated [3 marks]
- (ii) if ANY digit can be repeated. [2 marks]
- (b) Find the probability that a number in (a) (ii) above ends with the digit 9. [3 marks]
- (c) A farmer made three separate visits to the chicken farm to purchase chickens. On each visit he paid \$  $x$  for each grade A chicken, \$  $y$  for each grade B chicken and \$  $z$  for each grade C. His calculations are summarised in the table below.

Number of Visits	Number of Chickens Bought			Total Spent \$
	Grade A	Grade B	Grade C	
1	20	40	60	1 120
2	40	60	80	1 720
3	60	80	120	2 480

- (i) Use the information above to form a system of linear equations in  $x$ ,  $y$  and  $z$ . [3 marks]
- (ii) Express the system of equations in the form  $Ax = b$ . [2 marks]
- (iii) Solve the equations to find  $x$ ,  $y$  and  $z$ . [7 marks]

**Total 20 marks**

**END OF TEST**