FORM TP 02134032/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 03/B

1 *hour* 30 *minutes*

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Write your solutions, with full working, in the answer booklet provided.

4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2010**
Mathematical instruments
Silent, non-programmable electronic calculator

SECTION A (MODULE 1)

Answer this question.

1. (a) $z = 8 + (8\sqrt{3})\mathrm{i}$.

(i) Find the modulus and argument of z. **[3 marks]**

(ii) Using de Moivre's theorem show that z^3 is real, stating the value of z^3.

[2 marks]

(b) A complex number is represented by the point P in the Argand diagram.

(i) Given that $|z - 6| = |z|$ show that the locus of P is $x = 3$. **[2 marks]**

(ii) Find the complex numbers z which satisfy both

$$|z - 6| = |z| \text{ and } |z - 3 - 4\mathrm{i}| = 5.$$ **[5 marks]**

(c) Given $I_n = \int_0^8 x^n (8 - x)^{1/x}\, \mathrm{d}x, n \geq 0,$ show that

$$I_n = \frac{24n}{3n+4} I_{n-1}, n \geq 1.$$ **[8 marks]**

Total 20 marks

SECTION B (MODULE 2)

Answer this question

2. (a) (i) Show that $(r + 1)^3 - (r - 1)^3 = 6r^2 + 2$. **[2 marks]**

(ii) Hence show that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$. **[5 marks]**

(iii) Show that $\sum_{r=n}^{2n} r^2 = \frac{n}{6}(n+1)(an+b)$, where a and b are constants to be found.

[4 marks]

(b) The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0.$$

When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0.5$.

Find a Taylor series solution for x in ascending powers of t, up to and including the term in t^3.

[5 marks]

(c) Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

(i) Show that $2.2 < \alpha < 2.3$. **[2 marks]**

(ii) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

[2 marks]

Total 20 marks

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SECTION C (MODULE 3)

Answer this question

3. (a) Three identical cans of cola, two identical cans of green tea and two identical cans of orange juice are arranged in a row.

Calculate the number of arrangements if the first and last cans in the row are of the same type of drink.

[3 marks]

(b) Kris takes her dog for a walk every dy. The probability that they go to the park on any day is 0.6. If they go to the park there is a probability of 0.35 that the dog will bark. If they do not go to the park there is probability of 0.75 that the dog will bark.

Find the probability that the dog barks on any particular day. **[2 marks]**

(c) A committee of six people, which must consist of at least 4 men and at least one woman, is to be chosen from 10 men and 9 women.

Find the number of possible committees that include either Albert or Tracey but not both.

[3 marks]

(d) $A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$

(i) Find AB. **[2 marks]**

(ii) Deduce A^{-1}. **[2 marks]**

(iii) Given that $B^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix}$, prove that $(AB)^{-1} = B^{-1}A^{-1}$.

[2 marks]

(e) Find the general solution of the differential equation

$$\frac{dy}{dx} + y\cot x = \sin x.$$

[6 marks]

Total 20 marks

END OF TEST

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