FORM TP 2012235

TEST CODE 02234032

MAY/JUNE 2012

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

01 JUNE 2012 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

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SECTION A (Module 1)

Answer this question.

1. (a) Given that $y = \dfrac{x(x-1)^{\frac{1}{3}}}{1+\sin^3 x}$,

by taking logarithms of both sides, or otherwise, find $\dfrac{dy}{dx}$ in terms of x. **[4 marks]**

(b) (i) Sketch the curve $y = \sqrt{1+x^3}$, for values of x between $-\dfrac{1}{2}$ and 1. **[3 marks]**

(ii) Using the trapezium rule, with 5 intervals, find an approximation to

$$\int_0^1 \sqrt{1+x^3}\, dx.$$ **[5 marks]**

(c) (i) Use integration by parts to find

$$\int x^2 \cos x\, dx.$$ **[6 marks]**

(ii) Hence, find the area under the curve $y = x^2 \cos x$, between $x = 0$ and $x = \dfrac{\pi}{2}$.

[2 marks]

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) (i) Write down the binomial expansion of $\left(1+\frac{1}{4}x\right)^5$. **[4 marks]**

(ii) Hence, calculate $(1.025)^5$ correct to three decimal places. **[4 marks]**

(b) Let $f(x) = x^2 - 5x + 3$ and $g(x) = e^x$ be two functions.

(i) Sketch the graphs for $f(x)$ and $g(x)$ on the **same** coordinate axes for the domain $-1 \leq x \leq 2$. **[4 marks]**

(ii) Using $x_1 = 0.3$ as an initial approximation to the root x of $f(x) - g(x) = 0$, execute TWO iterations of the Newton-Raphson method to obtain a better approximation, x_3, of x **correct to four decimal places**. **[6 marks]**

(iii) Assuming that x_3 is the true root of $f(x) - g(x) = 0$, calculate the relative error of x_1. **[2 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) A computer programmer is trying to break into a company's code. His program generates a list of all permutations of any set of letters that it is given, without regard for duplicates. For example, given the letters **TTA**, it will generate a list of six 3-letter permutations (words).

If the program generates a list of all 8-letter permutations of **TELESTEL**, without regard for duplicates,

(i) how many times will any given word be repeated in the list? **[5 marks]**

(ii) in how many words will the first four letters be all different? **[5 marks]**

(b) (i) Find the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}.$$

[5 marks]

(ii) Find a 3×1 matrix, **Y**, such that

$$\mathbf{A} \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \mathbf{Y}.$$

[2 marks]

(iii) Hence, find a 3×3 matrix **B** such that

$$\mathbf{BY} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}.$$

[3 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.