C A R I B B E A N E X A M I N A T I O N S C O U N C I L

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 03/B

SOLUTIONS AND MARK SCHEMES

Question	Details	Marks
1 (a) (i)	$\|z\| = \sqrt{8^2 + (8\sqrt{3})^2} = 16$	1
	$\arg(z) = \pi - \tan^{-1}(\sqrt{3}) = \frac{2\pi}{3}$ (1) (1)	1 1
(ii)	$z^3 = 16^3 (\cos 2\pi/3 + i \sin 2\pi/3)^3$	1
	$= 16^3 (\cos 2\pi + i \sin 2\pi) = 4096$	1
(b) (i)	$(x-6)^2 + y^2 = x^2 + y^2$	1
	$x = 3$	1
(ii)	$\|z - 3 - 4i\| = 5$ is a circle with centre (3, 4) and radius 5	1
	Subs $x = 3$ gives $(3-3)^2 + (y-4)^2 = 5^2$	1
	$y^2 - 8y - 9 = 0 \quad (y-9)(y+1) = 0 \quad y = 9, -1$	1
	$z = 3 + 9i,\ 3 - i$ (1) (1)	1 1
(c)	$I_n = \frac{3}{4}\left[x^n (8-x)^{4/3}\right] + \frac{3}{4}\int nx^{n-1} (8-x)^{4/3}\,dx$ (1) (1)	1 1
	$= 0 + \frac{3}{4}\int nx^{n-1} (8-x)(8-x)^{1/3}\,dx$ (1) (1)	1 1
	$= \frac{3}{4}\int nx^{n-1}\, 8\, (8-x)^{1/3}\,dx - \frac{3}{4}\int nx^{n-1}\, x(8-x)^{1/3}\,dx$ (1) (1)	1 1
	$I_n = 6n\, I_{n-1} - \frac{3n}{4} I_n$	1
	$I_n = \frac{24n}{3n+4} I_{n-1}$	1
		20
S. O. (A) 7, 8, 11, 12, (C) 10		

Question	Details	Marks
2 (a)(i)	$r^3 + 3r^2 + 3r + 1 - (r^3 - 3r^2 + 3r - 1)$	1
	$= 6r^2 + 2$	1
(ii)	$r = 1$: $2^3 - 0^3 = 6(1)^2 + 2$ $r = 2$: $3^3 - 1^3 = 6(2)^2 + 2$ …………………………….. ……………………………..	1
	$r = n$: $(n + 1)^3 - (n - 1)^3 = 6n^2 + 2$	1
	summing gives $(n + 1)^3 + n^3 - 1 = 6\sum_1^n r^2 + 2n$ (1) (1)	1 1
	$\sum_{r=11}^{n} r^2 = \frac{n}{6}(n + 1)(2n + 1)$	1
(iii)	$\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$	1
	$= \frac{2n}{6}(2n + 1)(4n + 1) - \frac{n-1}{6}(n)(2n - 1)$ (1) (1)	1 1
	$= \frac{n}{6}(n + 1)(14n + 1)$	1
(b)	$f^{11}(t) = -x - \cos x$ $f^{11}(0) = -1$ (1) (1)	1 1
	$f^{111}(t) = (-1 + \sin x)\frac{dx}{dt}$ $f^{111}(0) = -0.5$ (1) (1)	1 1
	$f(t) = \frac{1}{2}t - \frac{1}{2}t^2 - \frac{1}{12}t^3 + \ldots$	1
(c) (i)	f(2.2) = -0.192 f(2.3) = 0.877 By IVT and continuity root exists (1) (1)	1 1
(ii)	$\frac{\alpha - 2.2}{0.192} = \frac{2.3 - \alpha}{0.877}$	1
	$\alpha = 2.218$	1
		20
S. O. (B) 1, 2, 4, 9, (D) 1, 3		

Question	Details	Marks
3 (a)	Ends cola: $\frac{5!}{2!2!} = 30$ ways	1
	Ends green tea: $\frac{5!}{3!2!} = 10$ ways	1
	Ends orange juice: $\frac{5!}{3!2!} = 10$ ways Total = 30 + 10 + 10 = 50 ways	1
(b)	P(bark) = P(park & bark) + P(no park & bark)	1
	= (0.6)(0.35) + (0.4)(0.75) = 0.51	1
(c)	Albert not Tracey = (9 C 3) × (8 C 2) + (9 C 4) × (8 C 1) = 3360	1
	Tracey not Albert = (9 C 4) × (8 C 1) + (9 C 5) = 1134	1
	# of selections = 3360 + 1134 = 4494	1
(d) (i)	$AB = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix} = \underset{(1)}{\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}} = \underset{(1)}{5I}$	1 1
(ii)	$AA^{-1}B = 5A^{-1}$	1
	$A^{-1} = \frac{1}{5}\begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$	1
(iii)	$(AB) = 5I \qquad (AB)(AB)^{-1} = 5(AB)^{-1}$	
	$(AB)^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	1
	$B^{-1}A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix} \times \frac{1}{5}\begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$	
	$= \frac{1}{5}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	1

Question	Details	Marks
3 (e)	$I = \mathrm{e}^{\int \cot x \, \mathrm{d}x}$	1
	$= \sin x$	1
	$\sin x \dfrac{\mathrm{d}y}{\mathrm{d}x} + y\cos x = \sin^2 x$	1
	$\int \dfrac{\mathrm{d}}{\mathrm{d}x}(y\sin x)\,\mathrm{d}x = \dfrac{1}{2}\int (1 - \cos 2x)\,\mathrm{d}x$	1
	(1) (1)	1
	$y \sin x = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$	1
		20
S. O. (A) 3, 4, 16, (B) 1, 2, 7, (C) 1		