

FORM TP 02234020/SPEC**CARIBBEAN EXAMINATIONS COUNCIL****ADVANCED PROFICIENCY EXAMINATION****PURE MATHEMATICS****UNIT 2****COMPLEX NUMBERS, ANALYSIS AND MATRICES****SPECIMEN PAPER****PAPER 02***2 hours 30 minutes*

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.

The maximum mark for each Module is 50.

The maximum mark for this examination is 150.

This examination consists of 5 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables

Electronic calculator

Ruler and graph paper

SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) (i) Express the complex number $\frac{4-2i}{1-3i}$ in the form of $a + ib$ where a and b are real numbers. [4 marks]
- (ii) Show that the argument of the complex number in (a) (i) above is $\frac{\pi}{4}$. [1 mark]
- (b) (i) Find the complex number $u = x + iy$, $x, y \in \mathbb{R}$ such that $u^2 = -5 + 12i$. [8 marks]
- (ii) Hence, solve the equation $z^2 + iz + (1 - 3i) = 0$. [6 marks]
- (c) Find the complex number $z = a + ib$ such that
- $$(1 + 3i)z + (4 - 2i)z = 10 + 4i$$
- [6 marks]

Total 25 marks

- 2 (a) Find $\int e^{3x} \sin 2x \, dx$ [7 marks]
- (b) (i)
- a) Find $\frac{dy}{dx}$ when $y = \tan^{-1}(3x)$. [4 marks]
- b) Hence, find $\int \frac{(x+2)}{1+9x^2} \, dx$. [4 marks]
- (ii) Show that if $y = \frac{\ln(5x)}{x^2}$ then $\frac{dy}{dx} = \frac{1 - \ln(25x^2)}{x^3}$ [5 marks]
- (c) Let $f(x, y) = x^2 + y^2 - 2xy$.
- (i) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ [2 marks]
- (ii) Show that $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2f(x, y)$ [3 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION B (MODULE 2)**Answer BOTH questions.**

3. (a) (i) Find constants A and B such that

$$\frac{1}{(2r-1)(2r+1)} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}. \quad [5 \text{ marks}]$$

- (ii) Hence, find the value of S where

$$S = \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)}. \quad [5 \text{ marks}]$$

- (iii) Deduce the sum to infinity of S . [3 marks]

- (b) (i) Find the r^{th} term of the series $1(2) + 2(5) + 3(8) + \dots$ [2 marks]

- (ii) Prove, by Mathematical Induction, that the sum to n terms of the series in (b) (i) above is $n^2(n+1)$. [10 marks]

Total 25 marks

4. (a) Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \dots$

- (i) show that the series is geometric [3 marks]

- (ii) find the sum of the series to n terms. [4 marks]

- (b) Use Maclaurin's Theorem to find the **first** three non-zero terms in the power series expansion of $\cos 2x$. [7 marks]

- (c) (i) Expand up to and including the term in x^3

$$\sqrt{\frac{1+x}{1-x}},$$

stating the values of x for which the expansion is valid. [7 marks]

- (ii) By taking $x = 0.02$ find an approximation for $\sqrt{51}$, correct to 5 decimal places.

[4 marks]**Total 25 marks**

GO ON TO THE NEXT PAGE

SECTION C (MODULE 3)**Answer BOTH questions.**

5. (a) Two cards are drawn without replacement from ten cards which are numbered 1 to 10. Find the probability that
- (i) the numbers on **BOTH** cards are even **[4 marks]**
- (ii) the number on one card is odd and the number on the other card is even. **[4marks]**

- (b) A journalist reporting on criminal cases classified 150 criminal cases by the age (in years) of the criminal and by the type of crime committed, violent or non-violent. The information is presented in the table below.

Type of Crime	Age (in years)		
	Less than 20	20 to 39	40 or older
Violent	27	41	14
Non-violent	12	34	22

What is the probability that a case randomly selected by the journalist

- (i) is a violent crime? **[2 marks]**
- (ii) was committed by someone LESS than 40 years old? **[4 marks]**
- (iii) is a violent crime OR was committed by a person LESS than 20 years old? **[3 marks]**
- (c) On a particular weekend, 100 customers made purchases at Green Thumb Garden supply store. Of these 100 customers;
- 30 purchased tools
 45 purchased fertilizer
 50 purchased seeds
 15 purchased seeds and fertilizer
 20 purchased seeds and tools
 15 purchased tools and fertilizer
 10 purchased tools, seeds and fertilizer.
- (i) Represent the above information on a Venn diagram. **[4 marks]**
- (ii) Determine how many customers purchased:
- a) only tools
 b) seeds and tools but not fertilizer,
 c) tools and fertilizer but not seeds,
 d) neither seeds, tools, nor fertilizer. **[4 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) Solve for x the following equation

$$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

[8 marks]

- (b) Solve the first order differential equation

$$y \tan x \frac{dy}{dx} = (4 + y^2) \sec^2 x$$

[4 marks]

- (c) Given that $y = u \cos 3x + v \sin 3x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = 30 \sin 3x,$$

find

- (i) the values of the constants u and v , **[8 marks]**
 (ii) the general solution of the differential equation. **[5 marks]**

Total 25 marks

END OF TEST