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FORM TP 2012234

MAY/JUNE 2012

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

25 MAY 2012 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 7 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A (Module 1)

Answer BOTH questions.

1. (a) (i) Given the curve $y = x^2 e^x$,

a) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **[5 marks]**

b) find the ***x*-coordinates** of the points at which $\frac{dy}{dx} = 0$ **[2 marks]**

c) find the ***x*-coordinates** of the points at which $\frac{d^2y}{dx^2} = 0$ **[2 marks]**

(ii) Hence , determine if the coordinates identified in (i) b) and c) above are at the maxima, minima or points of inflection of $y = x^2 e^x$. **[7 marks]**

(b) A curve is defined by the parametric equations $x = \sin^{-1} \sqrt{t}$, $y = t^2 - 2t$.

Find

(i) the gradient of a tangent to the curve at the point with parameter t **[6 marks]**

(ii) the equation of the tangent at the point where $t = \frac{1}{2}$. **[3 marks]**

Total 25 marks

2. (a) (i) Express

$$\frac{x^2 - 3x}{(x-1)(x^2+1)} \text{ in partial fractions.}$$ **[7 marks]**

(ii) Hence, find

$$\int \frac{x^2 - 3x}{x^3 - x^2 + x - 1}\, dx.$$ **[5 marks]**

(b) (i) Given that $\sin A \cos B - \cos A \sin B = \sin (A - B)$ show that

$$\cos 3x \sin x = \sin 3x \cos x - \sin 2x.$$ **[2 marks]**

(ii) If $I_m = \int \cos^m x \sin 3x \, dx$ and

$$J_m = \int \cos^m x \sin 2x \, dx,$$

prove that $(m + 3)\, I_m = mJ_{m-1} - \cos^m x \cos 3x$. **[7 marks]**

(iii) Hence, by putting $m = 1$, prove that

$$4\int_0^{\frac{\pi}{4}} \cos x \sin 3x \, dx = \int_0^{\frac{\pi}{4}} \sin 2x \, dx + \frac{3}{2}.$$ **[2 marks]**

(iv) Evaluate $\int_0^{\frac{\pi}{4}} \sin 2x \, dx$. **[2 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) For a particular G.P., $u_6 = 486$ and $u_{11} = 118\ 098$, where u_n is the n^{th} term.

(i) Calculate the first term, a, and the common ratio, r. **[5 marks]**

(ii) Hence, calculate n if $S_n = 177\ 146$. **[4 marks]**

(b) The first four terms of a sequence are $1 \times 3, 2 \times 4, 3 \times 5, 4 \times 6$.

(i) Express, in terms of r, the r^{th} term, u_r, of the sequence. **[2 marks]**

(ii) Prove, by mathematical induction, that

$$\sum_{r=1}^{n} u_r = \frac{1}{6} n\ (n+1)\ (2n+7), \forall n \in \mathbf{N}.$$ **[7 marks]**

(c) (i) Use Maclaurin's Theorem to find the first three non-zero terms in the power series expansion of $\cos 2x$. **[5 marks]**

(ii) Hence, or otherwise, obtain the first two non-zero terms in the power series expansion of $\sin^2 x$. **[2 marks]**

Total 25 marks

4. (a) (i) Express $\begin{pmatrix} n \\ r \end{pmatrix}$ in terms of factorials. **[1 mark]**

(ii) Hence, show that $\begin{pmatrix} n \\ r \end{pmatrix} = \begin{pmatrix} n \\ n-r \end{pmatrix}$. **[3 marks]**

(iii) Find the coefficient of x^4 in $\left(x^2 - \frac{3}{x}\right)^8$. **[5 marks]**

(iv) Using the identity $(1 + x)^{2n} = (1 + x)^n (1 + x)^n$, show that

$$\begin{pmatrix} 2n \\ n \end{pmatrix} = c_0^2 + c_1^2 + c_3^2 + \ldots + c_{n-1}^2 + c_n^2, \text{ where } c_r = \begin{pmatrix} n \\ r \end{pmatrix}.$$

[8 marks]

(b) Let $f(x) = 2x^3 + 3x^2 - 4x - 1 = 0$.

(i) Use the intermediate value theorem to determine whether the equation $f(x)$ has any roots in the interval [0.2, 2]. **[2 marks]**

(ii) Using $x_1 = 0.6$ as a first approximation of a root **T** of $f(x)$, execute FOUR iterations of the Newton–Raphson method to obtain a second approximation, x_2, of **T**. **[6 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C (Module 3)

Answer BOTH questions.

5. (a) How many 4-digit even numbers can be formed from the digits 1, 2, 3, 4, 6, 7, 8

(i) if each digit appears at most once? **[4 marks]**

(ii) if there is no restriction on the number of times a digit may appear? **[3 marks]**

(b) A committee of five is to be formed from among six Jamaicans, two Tobagonians and three Guyanese.

(i) Find the probability that the committee consists entirely of Jamaicans. **[3 marks]**

(ii) Find the number of ways in which the committee can be formed, given the following restriction: *There are as many Tobagonians on the committee as there are Guyanese.* **[6 marks]**

(c) Let **A** be the matrix $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

(i) Find the matrix **B**, where $\mathbf{B} = \mathbf{A}^2 - 3\mathbf{A} - \mathbf{I}$. **[3 marks]**

(ii) Show that $\mathbf{AB} = -9\mathbf{I}$. **[1 mark]**

(iii) Hence, find the inverse, $\mathbf{A}^{-1}$, of **A**. **[2 marks]**

(iv) Solve the system of linear equations

$$\mathbf{B}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}.$$

[3 marks]

Total 25 marks

6. (a) (i) Draw the points A and B on an Argand diagram,

$$\text{where } A = \frac{1+i}{1-i} \text{ and } B = \frac{\sqrt{2}}{1-i}.$$ **[6 marks]**

(ii) Hence, or otherwise, show that the argument of $\frac{(1+\sqrt{2}+i)}{1-i}$ is EXACTLY $\frac{3\pi}{8}$.

[5 marks]

(b) (i) Find ALL complex numbers, z, such that $z^2 = i$. **[3 marks]**

(ii) Hence, find ALL complex roots of the equation

$$z^2 - (3 + 5i)\,z - (4 - 7i) = 0.$$ **[5 marks]**

(c) Use de Moivre's theorem to show that

$$\cos 6\,\theta = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta.$$ **[6 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.