FORM TP 2006261

TEST CODE **02234020**

MAY/JUNE 2006

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

2 hours

31 MAY 2006 (p.m)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) If $f(x) = x^3 \ln^2 x$, show that

(i) $f'(x) = x^2 \ln x(3 \ln x + 2)$

[5 marks]

(ii) $f''(x) = 6x \ln^2 x + 10x \ln x + 2x$.

[5 marks]

(b) The enrolment pattern of membership of a country club follows an exponential logistic function N,

$$N = \frac{800}{1 + ke^{-rt}}, k \in \mathbf{R}, r \in \mathbf{R},$$

where N is the number of members enrolled t years after the formation of the club. The initial membership was 50 persons and after one year, there are 200 persons enrolled in the club.

(i) What is the LARGEST number reached by the membership of the club?

[2 marks]

(ii) Calculate the EXACT value of k and of r. [6 marks]

(iii) How many members will there be in the club 3 years after its formation?

[2 marks]

Total 20 marks

2. (a) (i) Express $\frac{1 + x}{(x-1)(x^2+1)}$ in partial fractions. [6 marks]

(ii) Hence, find $\int \frac{1 + x}{(x-1)(x^2+1)}\, dx$. [3 marks]

(b) Given that $I_n = \int_0^1 x^n e^x \, dx$, where $n \in N$.

(i) Evaluate I_1. [4 marks]

(ii) Show that $I_n = e - nI_{n-1}$. [4 marks]

(iii) Hence, or otherwise, evaluate I_3, writing your answer in terms of e.

[3 marks]

Total 20 marks

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Section B (Module 2)

Answer BOTH questions.

3. (a) (i) Show that the terms of

$$\sum_{r=1}^{m} \ln 3^r$$

are in arithmetic progression. **[3 marks]**

(ii) Find the sum of the first 20 terms of this series. **[4 marks]**

(iii) Hence, show that $\sum_{r=1}^{2m} \ln 3^r = (2m^2 + m) \ln 3$. **[3 marks]**

(b) The sequence of positive terms, $\{x_n\}$, is defined by $x_{n+1} = x_n^2 + \frac{1}{4}$, $x_1 < \frac{1}{2}$.

(i) Show, by mathematical induction, or otherwise, that $x_n < \frac{1}{2}$ for all positive integers n. **[7 marks]**

(ii) By considering $x_{n+1} - x_n$, or otherwise, show that $x_n < x_{n+1}$. **[3 marks]**

Total 20 marks

4. (a) Sketch the functions $y = \sin x$ and $y = x^2$ on the SAME axes. **[5 marks]**

(b) Deduce that the function $f(x) = \sin x - x^2$ has EXACTLY two real roots. **[3 marks]**

(c) Find the interval in which the non-zero root α of $f(x)$ lies. **[4 marks]**

(d) Starting with a first approximation of α at $x_1 = 0.7$, use one iteration of the Newton-Raphson method to obtain a better approximation of α to 3 decimal places. **[8 marks]**

Total 20 marks

Section C (Module 3)

Answer BOTH questions.

5. (a) (i) How many numbers lying between 3 000 and 6 000 can be formed from the digits, 1, 2, 3, 4, 5, 6, if no digit is used more than once in forming the number?
[5 marks]

(ii) Determine the probability that a number in 5 (a) (i) above is even.
[5 marks]

(b) In an experiment, p is the probability of success and q is the probability of failure in a single trial. For n trials, the probability of x successes and $(n-x)$ failures is represented by ${}^{n}C_{x}p^{x}q^{n-x}$, $n > 0$. Apply this model to the following problem.

The probability that John will hit the target at a firing practice is $\frac{5}{6}$. He fires 9 shots. Calculate the probability that he will hit the target

(i) AT LEAST 8 times **[7 marks]**

(ii) NO MORE than seven times. **[3 marks]**

Total 20 marks

6. (a) If $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 2 & 1 \\ 1 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(i) find AB **[3 marks]**

(ii) deduce A^{-1}. **[3 marks]**

(b) A nursery sells three brands of grass-seed mix, P, Q and R. Each brand is made from three types of grass, C, Z and B. The number of kilograms of each type of grass in a bag of each brand is summarised in the table below.

Grass Seed Mix	**Type of Grass (Kilograms)**		
	***C*-grass**	***Z*-grass**	***B*-grass**
Brand P	2	2	6
Brand Q	4	2	4
Brand R	0	6	4
Blend	c	z	b

A blend is produced by mixing p bags of Brand P, q bags of Brand Q and r bags of Brand R.

(i) Write down an expression in terms of p, q and r, for the number of kilograms of Z-grass in the blend. **[1 mark]**

(ii) Let c, z and b represent the number of kilograms of C–grass, Z–grass and B–grass respectively in the blend. Write down a set of THREE equations in p, q, r, to represent the number of kilograms of EACH type of grass in the blend. **[3 marks]**

(iii) Rewrite the set of THREE equations in (b) (ii) above in the matrix form $MX = D$ where M is a 3 by 3 matrix, X and D are column matrices. **[3 marks]**

(iv) Given that M^{-1} exists, write X in terms of M^{-1} and D. **[3 marks]**

(v) Given that $M^{-1} = \begin{pmatrix} -0.2 & -0.2 & 0.3 \\ 0.35 & 0.1 & -0.15 \\ -0.05 & 0.2 & -0.05 \end{pmatrix}$,

calculate how many bags of EACH brand, P, Q, and R, are required to produce a blend containing 30 kilograms of C-grass, 30 kilograms of Z-grass and 50 kilograms of B-grass. **[4 marks]**

Total 20 marks

END OF TEST