



CARIBBEAN EXAMINATIONS COUNCIL
ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

2 hours

01 JUNE 2005 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.

The maximum mark for each section is 40.

The maximum mark for this examination is 120.

This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables

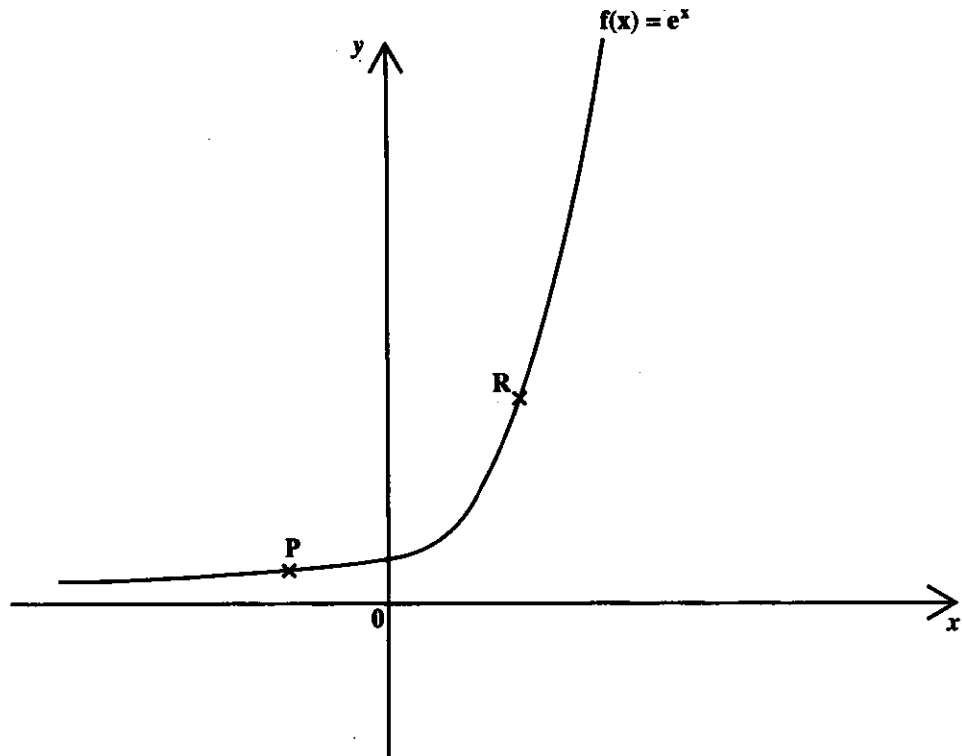
Electronic calculator

Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) The diagram below, not drawn to scale, shows two points, $P(p, 0.368)$ and $R(3.5, r)$, on $f(x) = e^x$ for $x \in \mathbf{R}$.



- (i) Copy the diagram above and on the same axes, sketch the graph of $g(x) = \ln x$.
[3 marks]
- (ii) Describe clearly the relationship between $f(x) = e^x$ and $g(x) = \ln x$.
[3 marks]
- (iii) Using a calculator, find the value of
- a) r [1 mark]
- b) p . [2 marks]
- (b) Given that
 $\log_a(bc) = x$, $\log_b(ca) = y$, $\log_c(ab) = z$ and $a \neq b \neq c$,
show that $a^x b^y c^z = (abc)^2$. [3 marks]
- (c) Find the values of $x \in \mathbf{R}$ for which $e^x + 3e^{-x} = 4$. [8 marks]

Total 20 marks

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2. (a) A curve is given parametrically by $x = (3 - 2t)^2$, $y = t^3 - 2t$. Find
- (i) $\frac{dy}{dx}$ in terms of t [4 marks]
- (ii) the gradient of the normal to the curve at the point $t = 2$. [2 marks]
- (b) (i) Express $\frac{2x+1}{x^2(x+1)}$ in the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$, where A, B and C are constants. [7 marks]
- (ii) Hence, evaluate $\int_1^2 \frac{2x+1}{x^2(x+1)} dx$. [7 marks]
- Total 20 marks**

Section B (Module 2)

Answer BOTH questions.

3. (a) (i) Use the fact that $\frac{1}{r} - \frac{1}{r+1} = \frac{1}{r(r+1)}$ to show that
- $$S_n = \sum_{r=1}^n \left(\frac{1}{r(r+1)} \right) = 1 - \frac{1}{n+1}.$$
- [5 marks]
- (ii) Deduce, that as $n \rightarrow \infty$, $S_n \rightarrow 1$. [1 mark]
- (b) The common ratio, r , of a geometric series is given by $r = \frac{5x}{4+x^2}$. Find ALL the values of x for which the series converges. [10 marks]
- (c) By substituting suitable values of x on both sides of the expansion of
- $$(1+x)^n = \sum_{r=0}^n {}^nC_r x^r,$$
- show that
- (i) $\sum_{r=0}^n {}^nC_r = 2^n$ [2 marks]
- (ii) $\sum_{r=0}^n {}^nC_r (-1)^r = 0$. [2 marks]
- Total 20 marks**

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4. The function, f , is given by $f(x) = 6 - 4x - x^3$.

(a) Show that

(i) f is everywhere strictly decreasing [4 marks]

(ii) the equation $f(x) = 0$ has a real root, α , in the closed interval $[1, 2]$ [4 marks]

(iii) α is the only real root of the equation $f(x) = 0$. [4 marks]

(b) If x_n is the n^{th} approximation to α , use the Newton-Raphson method to show that the $(n + 1)^{\text{st}}$ approximation x_{n+1} is given by

$$x_{n+1} = \frac{2x_n^3 + 6}{3x_n^2 + 4}. \quad [8 \text{ marks}]$$

Total 20 marks

Section C (Module 3)

Answer BOTH questions.

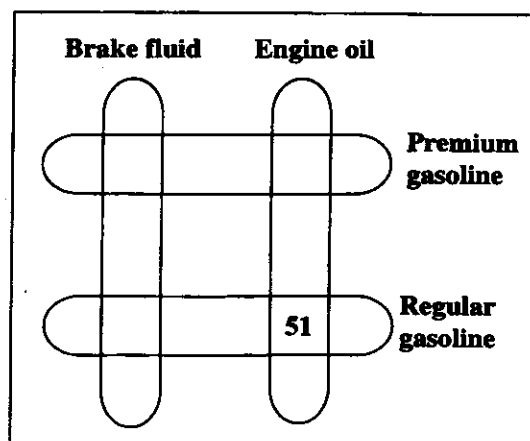
5. (a) On a particular day, a certain fuel service station offered 100 customers who purchased premium or regular gasoline, a free check of the engine oil or brake fluid in their vehicles. The services required by these customers were as follows:

15% of the customers purchased premium gasoline, the others purchased regular gasoline.

20% of the customers who purchased premium gasoline requested a check for brake fluid, the others requested a check for engine oil.

51 of the customers who purchased regular gasoline requested a check for engine oil, the others requested a check for brake fluid.

(i) Copy and complete the diagram below to represent the event space.



[3 marks]

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- (ii) Find the probability that a customer chosen at random
- a) who had purchased premium gasoline requested a check for engine oil
 - b) who had requested a check of the brake fluid purchased regular gasoline
 - c) who had requested a check of the engine oil purchased regular gasoline.
- [6 marks]

- (b) A bag contains 12 red balls, 8 blue balls and 4 white balls. Three balls are drawn from the bag at random **without replacement**.

Calculate

- (i) the total number of ways of choosing the three balls [3 marks]
- (ii) the probability that ONE ball of EACH colour is drawn [3 marks]
- (iii) the probability that ALL THREE balls drawn are of the SAME colour. [5 marks]

Total 20 marks

6. (a) Find the values of x for which

$$\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 2 & 1 & x \end{vmatrix} = 0.$$

[10 marks]

- (b) Twelve hundred people visited an exhibition on its opening day. Thereafter, the attendance fell each day by 4% of the number on the previous day.
- (i) Obtain an expression for the number of visitors on the n^{th} day. [2 marks]
 - (ii) Find the total number of visitors for the first n days. [3 marks]
 - (iii) The exhibition closed after 10 days. Determine how many people visited during the period for which it was opened. [3 marks]
 - (iv) If the exhibition had been kept opened indefinitely, what would be the maximum number of visitors? [2 marks]

Total 20 marks

END OF TEST