



**CARIBBEAN EXAMINATIONS COUNCIL**  
**ADVANCED PROFICIENCY EXAMINATION**  
**MATHEMATICS**  
**UNIT 2 – PAPER 03/2**

*$1\frac{1}{2}$  hours*

**24 MAY 2004 (p.m.)**

This examination paper consists of **THREE** questions, one question from each of Modules 2.1, 2.2 and 2.3.

The maximum mark for each question is 20.

The maximum mark for this examination is 60.

This examination paper consists of 4 pages.

**INSTRUCTIONS TO CANDIDATES**

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL THREE** questions.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

**Examination Materials**

Mathematical formulae and tables

Electronic calculator

Ruler and a pair of compasses

**SECTION A (MODULE 2.1)**

**Answer this question.**

1. (a) (i) Complete the table shown below for  $y = \frac{1}{2}(e^x + e^{-x})$  using a calculator where necessary. **[5 marks]**

$$y = \frac{1}{2}(e^x + e^{-x})$$

$x$	-2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	2.0
$e^x$	0.14	0.22		0.61		1.65		4.48	7.40
$e^{-x}$	7.40	4.48		1.65		0.61		0.22	0.14
$y$	3.27	2.35		1.13		1.13		2.35	3.27

- (ii) Hence, sketch the curve  $y = \frac{1}{2}(e^x + e^{-x})$  for values of  $x$  from -2 to +2. **[4 marks]**

- (b) Find  $\frac{dy}{dx}$ . **[2 marks]**

- (c) Show that  $1 + \left(\frac{dy}{dx}\right)^2 = y^2$ . **[4 marks]**

- (d) Show that  $\int_{-2}^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = e^2 - \frac{1}{e^2}$ . **[5 marks]**

**Total 20 marks**

**SECTION B (MODULE 2.2)**

**Answer this question.**

2. In a model for the growth of a population,  $p_n$  is the number of individuals in the population at the end of  $n$  years. Initially, the population consists of 1000 individuals.

In each calendar year, the population increases by 20%, and on December 31<sup>st</sup>, 100 individuals leave the population.

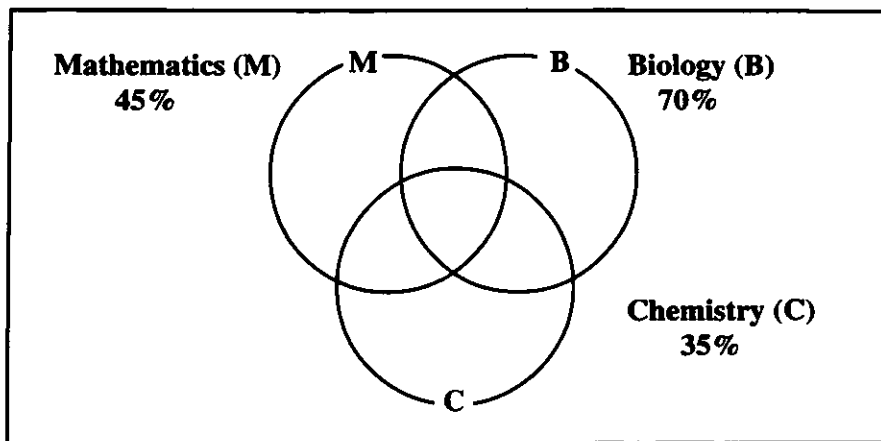
- (a) Calculate the values of  $p_1$  and  $p_2$ . **[4 marks]**
- (b) Write down an equation connecting  $p_{n+1}$  and  $p_n$ . **[2 marks]**
- (c) Show by Mathematical Induction, or otherwise, that  $p_n = 500 (1.2)^n + 500$ . **[9 marks]**
- (d) Calculate the **smallest** value of  $n$  for which  $p_n > 10\,000$ . **[5 marks]**

**Total 20 marks**

**SECTION C (MODULE 2.3)**

**Answer this question.**

3. (a) In a sixth form, the students are studying one or more of the subjects, Biology (B), Chemistry (C) and Mathematics (M). 15% of the students are studying both Chemistry and Mathematics and 3% of them are studying all three subjects. Some of this information is shown on the diagram below.



- (i) What is the probability that a student chosen at random is studying Chemistry? [1 mark]
- (ii) Given that a student is studying Chemistry, what is the probability that the student is also studying Mathematics? [3 marks]
- (iii) Find the probability that a student who is studying Chemistry and Mathematics is also studying Biology. [3 marks]
- (b)  $A$  and  $B$  are two independent events such that  $P(A) = \frac{1}{3}$  and  $P(A \cap B) = \frac{1}{4}$ . Find
- (i)  $P(B)$  [4 marks]
- (ii)  $P(A \cup B)$ . [4 marks]
- (c) A three-digit number is formed by choosing, with replacement, three digits at random from the digits 1, 2, 3, 4, 5. What is the probability that the number formed is divisible by 5? [5 marks]

**Total 20 marks**

**END OF TEST**