CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

2 hours

30 MAY 2007 (p.m)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) Solve, for $x > 0$, the equation $3 \log_8 x = 2 \log_x 8 - 5$. **[8 marks]**

(b) (i) Copy and complete the table below for values 2^x and e^{-x} using a calculator, where necessary. **Approximate all values to 2 decimal places.**

x	−1.0	0	0.5	1.0	1.5	2.0	2.5	3.0
2^x		1.00	1.41	2.00		4.00		8.00
e^{-x}	2.72			0.37	0.22			0.05

[3 marks]

(ii) On the same pair of axes and using a scale of 4 cm for 1 unit on the x-axis, 4 cm for 1 unit on the y-axis, draw the graphs of the two curves $y = 2^x$ and $y = e^{-x}$ for $-1 \leq x \leq 3, x \in \mathbf{R}$. **[5 marks]**

(iii) Use your graphs to find

a) the value of x satisfying $2^x - e^{-x} = 0$ **[2 marks]**

b) the range of values of x for which $2^x - e^{-x} < 0$. **[2 marks]**

Total 20 marks

2. (a) Show that for $n \geq 2$, $\tan^n x = \tan^{n-2} x \sec^2 x - \tan^{n-2} x$. **[3 marks]**

(b) Find $\frac{dy}{dx}$ when $y = \tan^n x$. **[3 marks]**

(c) Let $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx, n \geq 2$.

(i) By using the result in (a) above, show that $I_n + I_{n-2} = \frac{1}{n-1}$. **[7 marks]**

(ii) Hence evaluate I_4. **[7 marks]**

Total 20 marks

Section B (Module 2)

Answer BOTH questions.

3. (a) The sequence $\{u_n\}$ is given by $u_1 = 1$ and $u_{n+1} = (n+1)\,u_n$, $n \geq 1$.

Prove by Mathematical Induction that $u_n = n!\ \ \forall n \in \mathbf{N}$. **[9 marks]**

(b) Given that the sum of the first n terms of a series, S, is $9 - 3^{2-n}$,

(i) find the n-th term of S **[5 marks]**

(ii) show that S is a geometric progression **[2 marks]**

(iii) find the first term and common ratio of S **[2 marks]**

(iv) deduce the sum to infinity of S. **[2 marks]**

Total 20 marks

4. (a) The function f is given by $f: x \rightarrow x^4 - 4x + 1$. Show that

(i) $f(x) = 0$ has a root α in the interval $(0, 1)$ **[4 marks]**

(ii) if x_1 is a first approximation to α of $f(x) = 0$ in $(0, 1)$, the Newton-Raphson method gives a second approximation x_2 in $(0, 1)$ satisfying $x_2 = \dfrac{3x_1^{\,4} - 1}{4\left(x_1^{\,3} - 1\right)}$. **[5 marks]**

(b) John's father gave him a loan of \$10 800 to buy a car. The loan was to be repaid by 12 unequal monthly instalments, starting with an initial payment of \$$P$ in the first month. There is no interest charged on the loan, but the instalments increase by \$60 per month.

(i) Show that $P = 570$. **[5 marks]**

(ii) Find, in terms of n, $1 \leq n \leq 12$, an expression for the remaining debt on the loan after John has paid the n-th instalment. **[6 marks]**

Total 20 marks

Section C (Module 3)

Answer BOTH questions.

5. (a) A bag contains 5 white marbles and 5 black marbles. Six marbles are chosen at random.

(i) Determine the number of ways of selecting the six marbles if there are no restrictions. **[2 marks]**

(ii) Find the probability that the marbles chosen contain more black marbles than white marbles. **[4 marks]**

(b) The table below summarises the programme preference of 100 television viewers.

Television Preference	Number of Males	Number of Females	Total
Matlock	20	10	30
News	14	18	32
Friends	18	20	38
Total	52	48	100

Determine the probability that a person selected at random

(i) is a female **[2 marks]**

(ii) is a male or likes watching the News **[4 marks]**

(iii) is a female that likes watching Friends **[2 marks]**

(iv) does not like watching Matlock. **[2 marks]**

(c) The table below lists the probability distribution of the number of accidents per week on a particular highway.

Number of Accidents Per Week	0	1	2	3	4	5
Probability	0.25	0	0.10	p	0.30	0.15

(i) Calculate the value of p. **[2 marks]**

(ii) Determine the probability that there are more than 3 accidents in a week. **[2 marks]**

Total 20 marks

6. (a) A system of equations is given by

$$x + y + z = 10$$
$$3x - 2y + 3z = 35$$
$$2x + y + 2z = \alpha$$

where α is a real number.

(i) Write the system in matrix form. **[1 mark]**

(ii) Write down the augmented matrix. **[1 mark]**

(iii) Reduce the augmented matrix to echelon form. **[3 marks]**

(iv) Deduce the value of α for which the system is consistent. **[1 mark]**

(v) Find **ALL** solutions corresponding to this value of α. **[4 marks]**

(b) Given $A = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$,

find

(i) $kI - A$, where I is the 3 x 3 Identity matrix and k is a real number **[3 marks]**

(ii) the values of k for which $|\, kI - A \,| = 0$.

[7 marks]

Total 20 marks

END OF TEST