CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 2 – PAPER 02

$2\frac{1}{2}$ hours

02 JUNE 2004 (p.m.)

This examination paper consists of THREE sections: Module 2.1, Module 2.2 and Module 2.3.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

SECTION A (MODULE 2.1)

Answer BOTH questions.

1. (a) If $2\log_a 2 + \log_a 10 - 3\log_a 3 = 3 + \log_a 5$, $a > 0$, find the value of a. **[5 marks]**

(b) Find the value(s) of $x \in \mathbb{R}$ which satisfy $2\log_3 x = \log_3(x+6)$. **[5 marks]**

(c) Complete the table below for values of 2^x and e^x using a calculator, where necessary. Approximate all values to 1 decimal place.

x	-2	-1	0	0.5	1	1.5	2	2.5	3
2^x	0.3			1.4		2.8		5.7	
e^x	0.1			1.6		4.5		12.2	

[4 marks]

(d) On the same pair of axes and using a scale of 2 cm for 1 unit on the x-axis, 1 cm for 1 unit on the y-axis, draw the graphs of the two curves
$y = 2^x$ and $y = e^x$ for $-2 \le x \le 3$. **[7 marks]**

(e) Use your graphs to find

(i) the value of x satisfying $2^x = e^x$ **[2 marks]**

(ii) the SMALLEST INTEGER x for which $e^x - 2^x > 3$. **[2 marks]**

Total 25 marks

2. (a) Differentiate, with respect to x, EACH function below. Simplify your answers as far as possible.

(i) $\dfrac{e^x}{x+1}$ **[5 marks]**

(ii) $\tan^2(x^3)$. **[5 marks]**

(b) Use the substitution $u = \sin x$ to find

$\int e^{\sin x} \cos x \, dx$. **[5 marks]**

(c) The parametric equations of a curve are given by
$x = 3 - 2t, \; y = t(1-t)$.

(i) Find $\dfrac{dy}{dx}$ in terms of t. **[4 marks]**

(ii) A normal to the curve is parallel to the line $x + y = 2$. Find the equation of this normal. **[6 marks]**

Total 25 marks

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SECTION B (MODULE 2.2)

Answer BOTH questions.

3. (a) A sequence of real numbers $\{u_n\}$ satisfies the recurrence relation:

$$u_1 = 1 \,,\; u_n u_{n+1} = 2 \,.$$

(i) Show that $u_{n+2} = u_n$. **[2 marks]**

(ii) Given that $a_n = u_{n+1} + u_n$ and $b_n = u_{n+1} - u_n$, write down the first FOUR terms of each of the sequences $\{u_n\}$, $\{a_n\}$ and $\{b_n\}$. **[6 marks]**

(iii) State which of the sequences in (ii) above is convergent, divergent or periodic. **[3 marks]**

(b) Prove by mathematical induction that

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}\, n(n+1)(n+2) \text{, for all } n \in \mathbf{N}.$$ **[9 marks]**

(c) Find the sum of the arithmetic progression
72, 69, 66, ..., -24, -27. **[5 marks]**

Total 25 marks

4. (a) (i) The function $f: \boldsymbol{R} \to \boldsymbol{R}$ is given by $f(x) = x^3 + 2x - 2$.
Show that

a) f is a strictly increasing function **[3 marks]**

b) the equation $f(x) = 0$ has a root α in the interval [0, 1] **[3 marks]**

c) the equation $f(x) = 0$ has **no other root** in the interval [0, 1]. **[3 marks]**

(ii) By starting with $x_1 = 0.5$ as a first approximation to the root, α, use the Newton-Raphson method to find a second approximation, x_2, to the root α correct to 3 decimal places. **[4 marks]**

(b) Given that the coefficient of x^2 is zero in the binomial expansion of $(1 - ax)(1 + 2x)^5$, find the value of a and the coefficient of x^3. **[12 marks]**

Total 25 marks

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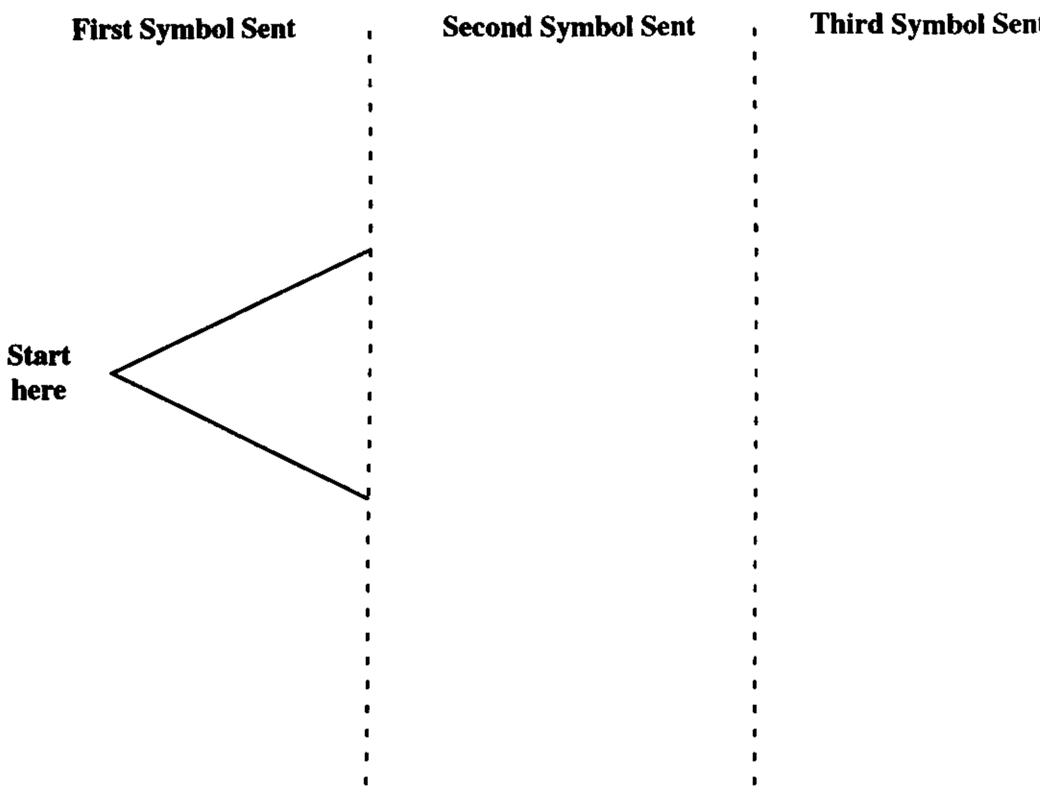
SECTION C (MODULE 2.3)

Answer BOTH questions.

5. (a) A message is sent using two symbols, α and β, arranged in sequence. The probability that the first symbol sent is α, is $\frac{1}{5}$.
A tree diagram is started in the diagram below.



Copy the diagram in your answer booklet and use the information above to write the correct probability on EACH branch of the tree diagram. **[2 marks]**

(b) After the first symbol has been sent, the probability that an α is sent is $\frac{1}{4}$ if the preceding symbol was an α and $\frac{1}{3}$ if the preceding symbol was a β.

(i) Use this information to extend your diagram to represent the FIRST THREE symbols sent. **[3 marks]**

(ii) Write CLEARLY the probabilities on EACH branch of your tree diagram. **[8 marks]**

(c) Using the information from your tree diagram, find the probability that

(i) there will be EXACTLY TWO α's among the FIRST THREE symbols sent **[6 marks]**

(ii) the THREE symbols sent are identical. **[6 marks]**

Total 25 marks

6. (a) A rectangular container with a lid, is made from thin metal. Its length is $2x$ metres and its width is x metres. The box must have a volume of 72 cubic metres.

(i) Show that the area, A square metres, of metal used is given by $A = 4x^2 + \dfrac{216}{x}$. **[5 marks]**

(ii) Find the value of x so that A is a minimum. **[5 marks]**

(b) The cost of making x articles per day is $\$(\frac{1}{2}x^2 + 50x + 50)$ and the selling price of each article is $\$(80 - \frac{1}{4}x)$.
Find

(i) the daily profit in terms of x **[5 marks]**

(ii) the value of x to give a maximum profit **[3 marks]**

(iii) the maximum value of the profit. **[2 marks]**

(c) A plot of land is rented on the understanding that the rent for the first year will be \$64 and in subsequent years will always be $\frac{7}{8}$ of what it was the year before. Calculate, **to the nearest dollar**, the TOTAL amount of rent paid for the first 15 years. **[5 marks]**

Total 25 marks

END OF TEST