FORM TP 2008243

TEST CODE **02234020**

MAY/JUNE 2008

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 ½ hours

28 MAY 2008 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 5 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Differentiate with respect to x

   (i) $e^{4x} \cos \pi x$ **[4 marks]**

   (ii) $\ln \dfrac{x^2 + 1}{\sqrt{x}}$. **[4 marks]**

   (b) Given $y = 3^{-x}$, show, by using logarithms, that

   $$\frac{dy}{dx} = -3^{-x} \ln 3.$$ **[5 marks]**

   (c) (i) Express in partial fractions

   $$\frac{2x^2 - 3x + 4}{(x-1)(x^2+1)}.$$ **[7 marks]**

   (ii) Hence, find

   $$\int \frac{2x^2 - 3x + 4}{(x-1)(x^2+1)} \, dx.$$ **[5 marks]**

Total 25 marks

2. (a) Solve the differential equation

   $$\frac{dy}{dx} + y = e^{2x}.$$ **[5 marks]**

   (b) The gradient at the point (x, y) on a curve is given by

   $$\frac{dy}{dx} = e^{4x}.$$

   Given that the curve passes through the point (0, 1), find its equation. **[5 marks]**

   (c) Evaluate $\int_1^e x^2 \ln x \, dx$, writing your answer in terms of e. **[7 marks]**

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(d) (i) Use the substitution $v = 1 - u$ to find

$$\int \frac{du}{\sqrt{1 - u}}.$$ **[3 marks]**

(ii) Hence, or otherwise, use the substitution $u = \sin x$ to evaluate

$$\int_0^{\pi/2} \sqrt{1 + \sin x} \quad dx.$$ **[5 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) A sequence $\{u_n\}$ is defined by the recurrence relation

$$u_{n+1} = u_n + n, \; u_1 = 3, \quad n \in \mathbf{N}.$$

(i) State the first FOUR terms of the sequence. **[3 marks]**

(ii) Prove by mathematical induction, or otherwise, that

$$u_n = \frac{n^2 - n + 6}{2}.$$ **[8 marks]**

(b) A GP with first term a and common ratio r has sum to infinity 81 and the sum of the first four terms is 65. Find the values of a and r. **[6 marks]**

(c) (i) Write down the first FIVE terms in the power series expansion of ln $(1 + x)$, stating the range of values of x for which the series is valid. **[3 marks]**

(ii) a) Using the result from (c) (i) above, obtain a similar expansion for ln $(1 - x)$. **[2 marks]**

b) Hence, prove that

$$\ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \ldots\right).$$ **[3 marks]**

Total 25 marks

4. (a) (i) Show that the function $f(x) = x^3 - 3x + 1$ has a root α in the closed interval $[1, 2]$. **[3 marks]**

(ii) Use the Newton-Raphson method to show that if x_1 is a first approximation to α in the interval $[1, 2]$, then a second approximation to α in the interval $[1, 2]$ is given by

$$x_2 = \frac{2x_1^3 - 1}{3x_1^2 - 3}.$$ **[5 marks]**

(b) (i) Use the binomial theorem or Maclaurin's theorem to expand $(1+x)^{-\frac{1}{2}}$ in ascending powers of x **as far as the term in x^3**, stating the values of x for which the expansion is valid. **[4 marks]**

(ii) Obtain a similar expansion for $(1 - x)^{\frac{1}{2}}$. **[4 marks]**

(iii) Prove that if x is so small that x^3 and higher powers of x can be neglected, then

$$\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2.$$ **[5 marks]**

(iv) Hence, by taking $x = \frac{1}{17}$, show, **without using calculators or tables**, that $\sqrt{2}$ is approximately equal to $\frac{1635}{1156}$. **[4 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) A cricket selection committee of 4 members is to be chosen from 5 former batsmen and 3 former bowlers.

In how many ways can this committee be selected so that the committee includes AT LEAST

(i) ONE former batsman? **[8 marks]**

(ii) ONE batsman and ONE bowler? **[3 marks]**

(b) Given the matrices

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 3 & -1 \end{pmatrix} \text{ and } \mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -3 \\ -1 & 3 & -1 \end{pmatrix},$$

(i) determine EACH of the following matrices:

a) $\mathbf{A} - \mathbf{B}$ **[2 marks]**

b) $\mathbf{AM}$ **[3 marks]**

(ii) deduce from (i) b) above the inverse $\mathbf{A}^{-1}$ of the matrix **A** **[3 marks]**

(iii) find the matrix **X** such that $\mathbf{AX} + \mathbf{B} = \mathbf{A}$. **[6 marks]**

Total 25 marks

6. (a) (i) Express the complex number

$\dfrac{2-3i}{5-i}$ in the form $\lambda\,(1-i)$. **[4 marks]**

(ii) State the value of λ. **[1 mark]**

(iii) Verify that $\left(\dfrac{2-3i}{5-i}\right)^4$ is a real number and state its value. **[5 marks]**

(b) The complex number z is represented by the point T in an Argand diagram.

Given that $z = \dfrac{1}{3+it}$ where t is a variable and $\bar{z}$ denotes the complex conjugate of z, show that

(i) $z + \bar{z} = 6\,z\bar{z}$ **[7 marks]**

(ii) as t varies, T lies on a circle, and state the coordinates of the centre of this circle. **[8 marks]**

Total 25 marks

END OF TEST