



CARIBBEAN EXAMINATIONS COUNCIL  
ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

$1\frac{1}{2}$  hours

22 MAY 2006 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2, and Module 3.

Each section consists of 1 question.

The maximum mark for each section is 20.

The maximum mark for this examination is 60.

This examination paper consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables

Electronic calculator

Graph paper

**Section A (Module 1)**

**Answer this question.**

1. The rate of increase of the number of algae with respect to time,  $t$  days, is equal to  $k$  times  $f(t)$ , where  $f(t)$  is the number of algae at any given time  $t$  and  $k \in \mathbf{R}$ .
- (a) Obtain a differential equation involving  $f(t)$  which may be used to model this situation. [ 1 mark ]
- (b) Given that
- the number of algae at the beginning is  $10^6$
  - the number of algae doubles every 2 days,
- (i) determine the values of  $f(0)$  and  $f(2)$  [ 2 marks]
- (ii) show that
- a)  $k = \frac{1}{2} \ln 2$  [10 marks]
  - b)  $f(t) = 10^6(2^{t/2})$  [ 5 marks]
- (iii) determine the approximate number of algae present after 7 days. [ 2 marks]

**Total 20 marks**

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**Section B (Module 2)**

**Answer this question.**

2. (a) A car was purchased at the beginning of the year, for  $P$  dollars. The value of a car at the end of each year is estimated to be the value at the beginning of the year multiplied by  $(1 - \frac{1}{q})$ ,  $q \in N$ .

- (i) Copy and complete the table below showing the value of the car for the first five years after purchase.

	Year 1	Year 2	Year 3	Year 4	Year 5
<b>Value at the Beginning of Year (\$)</b>	$P$	$P(1 - \frac{1}{q})$	$P(1 - \frac{1}{q})^2$		
<b>Value at the End of Year (\$)</b>	$P(1 - \frac{1}{q})$	$(1 - \frac{1}{q}) \left[ P(1 - \frac{1}{q}) \right]$ $= P(1 - \frac{1}{q})^2$			

[ 3 marks]

- (ii) Describe FULLY the sequence shown in the table. [ 2 marks]
- (iii) Determine, in terms of  $P$  and  $q$ , the value of the car  $n$  years after purchase. [ 1 mark ]
- (b) If the original value of the car was \$20 000.00 and the value at the end of the fourth year was \$8 192.00, find
- (i) the value of  $q$  [ 5 marks]
- (ii) the estimated value of the car after five years [ 2 marks]
- (iii) the LEAST integral value of  $n$ , the number of years after purchase, for which the estimated value of the car falls below \$500.00. [ 7 marks]

**Total 20 marks**

**Section C (Module 3)**

**Answer this question.**

3. (a) A box contains 8 green balls and 6 red balls. Five balls are selected at random. Find the probability that
- (i) ALL 5 balls are green [ 4 marks]
  - (ii) EXACTLY 3 of the five balls are red [ 4 marks]
  - (iii) at LEAST ONE of the five balls is red. [ 3 marks]
- (b) Use the method of row reduction to echelon form on the augmented matrix for the following system of equations to show that the system is inconsistent. [ 9 marks]

$$\begin{aligned}x + 2y + 4z &= 6 \\y + 2z &= 3 \\x + y + 2z &= 1\end{aligned}$$

**Total 20 marks**

**END OF TEST**