



CARIBBEAN EXAMINATIONS COUNCIL  
ADVANCED PROFICIENCY EXAMINATION

## PURE MATHEMATICS

## UNIT 2 – PAPER 03/B

## ANALYSIS, MATRICES AND COMPLEX NUMBERS

*1 ½ hours*

19 MAY 2008 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 4 printed pages.

**INSTRUCTIONS TO CANDIDATES**

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

**Examination Materials Permitted**

Graph paper (provided)

Mathematical formulae and tables (provided) – Revised 2008

Mathematical instruments

Silent, non-programmable, electronic calculator

**SECTION A (Module 1)**

**Answer this question.**

1. (a) The parametric equations of a curve are given by  $x = 3t^2$  and  $y = 6t$ .
- (i) Find the value of  $\frac{dy}{dx}$  at the point  $P$  on the curve where  $y = 18$ . [5 marks]
  - (ii) Find the equation of the normal to the curve at  $P$ . [3 marks]
- (b) In an experiment it was discovered that the volume,  $V \text{ cm}^3$ , of a certain substance in a room after  $t$  seconds may be determined by the equation
- $$V = 60 e^{0.04t}$$
- (i) Find  $\frac{dV}{dt}$  in terms of  $t$ . [3 marks]
  - (ii) Determine the rate at which the volume
    - a) increases after 10 seconds [1 mark]
    - b) is increasing when it is  $180 \text{ cm}^3$ . [3 marks]
  - (iii) Sketch the graph of  $V = 60 e^{0.04t}$  showing the point(s) of intersection, where they exist, with the axes. [5 marks]

**Total 20 marks**

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**SECTION B (Module 2)**

**Answer this question.**

2. (a) Matthew started a savings account at a local bank by depositing \$5 in the first week. In each succeeding week after the first, he added twice the amount deposited in the previous week.

(i) Derive an expression for

a) the amount deposited in the  $r^{\text{th}}$  week, in terms of  $r$  [3 marks]

b) the TOTAL amount in the account after  $n$  weeks, in terms of  $n$ . [3 marks]

(ii) Calculate the MINIMUM number,  $n$ , of weeks it would take for the amount in the account to exceed \$1000.00 if no withdrawal is made. [3 marks]

(b) The series  $S$  is given by

$$S = 1 \frac{1}{2} + 3 \frac{1}{4} + 5 \frac{1}{8} + 7 \frac{1}{16} + \dots$$

(i) Express  $S$  as the sum of an  $AP$  and a  $GP$ . [3 marks]

(ii) Find the sum of the first  $n$  terms of  $S$ . [3 marks]

(c) (i) Use the binomial theorem to expand  $\frac{1}{1-y}$  as a power series in  $y$  as far as the term in  $y^4$ . [2 marks]

(ii) Given that the Maclaurin series expansion for  $\cos x$  is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

find the first THREE non-zero terms in the power series expansion of  $\sec x$ .

[3 marks]

**Total 20 marks**

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**SECTION C (Module 3)**

**Answer this question.**

3. (a) (i) By considering the augmented matrix for the following system of equations, determine the value of  $k$  for which the system is consistent.

$$x + 3y + 5z = 2$$

$$x + 4y - z = 1$$

$$y - 6z = k$$

**[5 marks]**

- (ii) Find ALL the solutions to the system for the value of  $k$  obtained in (i) above. **[4 marks]**

- (b) The probability that a person selected at random

- owns a car is 0.25
- is self-employed is 0.40
- is self-employed OR owns a car is 0.6.

- (i) Determine the probability that a person selected at random owns a car AND is self-employed. **[4 marks]**

- (ii) Stating a reason in EACH case, determine whether the events 'owns a car' and 'is self-employed' are

a) independent events **[4 marks]**

b) mutually exclusive events. **[3 marks]**

**Total 20 marks**

**END OF TEST**

