

FORM TP 2009238



TEST CODE **02234032**

MAY/JUNE 2009

**CARIBBEAN EXAMINATIONS COUNCIL
ADVANCED PROFICIENCY EXAMINATION**

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

03 JUNE 2009 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 3 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)

Mathematical formulae and tables (provided) – Revised 2009

Mathematical instruments

Silent, non-programmable, electronic calculator

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02234032/CAPE 2009

SECTION A (Module 1)

Answer this question.

1. (a) Solve the differential equation

$$\frac{dy}{dx} + \frac{y}{x(x+1)} = (x+1)e^{-x^2}. \quad [7 \text{ marks}]$$

- (b) A curve is being cut by an automatic machine. The x and y coordinates of the curve are connected by the differential equation

$$\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} - 4y = 5 \sin x + 3 \cos x.$$

Find the equation of the curve, given that the curve passes through the origin and that $y = e^{-x} - e^{4x}$ when $x = \pi$. [13 marks]

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) Prove by mathematical induction that

$$\sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4}$$

for all integers $n \geq 1$. [7 marks]

- (b) An A.P. with ten terms has first term 60 and last term -120 . Find the sum of ALL the terms. [4 marks]

GO ON TO THE NEXT PAGE

- (c) John's starting annual salary at a company is \$25 000. His contract at the company states that his annual salary in subsequent years will be increased by 2% over the salary of the previous year.

Find, to the nearest dollar,

- (i) John's salary for the tenth year with the company [4 marks]
- (ii) the TOTAL amount of money which the company would have paid to John at the end of his first ten years with the company. [5 marks]

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) There are 6 staff members and 7 students on the sports council of a college. A committee of 10 persons is to be selected to organize a tournament. Calculate the number of ways in which the committee can be selected if the number of students must be greater than or equal to the number of staff members. [6 marks]

- (b) A and B are events such that

$$P(A) = 0.6, \quad P(B) = 0.2 \quad \text{and} \quad P(A \cap B) = 0.1.$$

Calculate

- (i) $P(A \cup B)$ [2 marks]
- (ii) $P(A \cap B')$ [2 marks]
- (iii) the probability that exactly ONE of A and B will occur. [4 marks]
- (c) (i) Show that the locus of the complex number z such that

$$|z + i - 1| = 5$$

is a circle. [4 marks]

- (ii) Find the centre and radius of the circle in (c) (i) above. [2 marks]

Total 20 marks

END OF TEST