TEST CODE **02234032**

FORM TP 2011235

MAY/JUNE 2011

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

01 JUNE 2011 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 3 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2010**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this question.

1. (a) A target is moving along a curve whose parametric equations are

$$x = 4 - 3\cos t,\ \ y = 5 + 2\sin t,$$

where t is the time. The distances are measured in metres.

Let θ be the angle which the tangent to the curve makes with the positive x-axis.

(i) Find the rate at which θ is increasing or decreasing when $t = \frac{2\pi}{3}$ seconds. **[7 marks]**

(ii) What are the units of the rate of increase? **[1 mark]**

(iii) Find the Cartesian equation of the curve. **[2 marks]**

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 8x^2.$$ **[10 marks]**

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) (i) Show that the equation $x^2 + 8x - 8 = 0$ has a root, α, in the interval $[0, 1]$. **[3 marks]**

(ii) By taking $x_0 = 0$ as the first approximation for α and using the formula $x_{n+1} = \frac{8 - x_n^{\ 2}}{8}$ **three** times, find a better approximation for α. **[3 marks]**

(b) (i) Write down the **first** FOUR non-zero terms of the expansions of $\ln(1 - x)$ and e^{-x} in ascending powers of x, stating for EACH expansion the range of values of x for which it is valid. **[3 marks]**

(ii) If $-1 \leq x < 1$ and $y = x + \frac{x^2}{2} + \frac{x^3}{3} + ...$, prove that

$$x = 1 - e^{-y}.$$ **[2 marks]**

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(c) In a model for the growth of a population, p_n is the number of individuals in the population **at the end** of n years. Initially, the population consists of 1000 individuals. In EACH calendar year (January to December), the population increases by 20% and on 31 December, 100 individuals leave the population.

(i) Calculate the values of p_1 and p_2. **[2 marks]**

(ii) Obtain an equation connecting p_{n+1} and p_n. **[1 mark]**

(iii) Show that $p_n = 500(1.2)^n + 500$. **[6 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) Let $\mathbf{A} = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$

(i) Show that $\mathbf{A}^2 - 3\mathbf{A} + 2\mathbf{I} = 0$. **[6 marks]**

(ii) Deduce that $\mathbf{A}^{-1} = \frac{1}{2}(3\mathbf{I} - \mathbf{A})$. **[4 marks]**

(iii) Hence, find the solution of the system of equations

$$5x - 6y - 6z = 10$$
$$-x + 4y + 2z = -4$$
$$3x - 6y - 4z = 8.$$

[3 marks]

(b) If $z = \frac{2+i}{1-i}$, find the real and imaginary parts of $z + \frac{1}{z}$. **[4 marks]**

(c) If $z + \frac{1}{z}$ is written in the form $r(\cos\theta + i\sin\theta)$ where r is the real and positive, find r and $\tan\theta$. **[3 marks]**

Total 20 marks

END OF TEST