CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

$1\frac{1}{2}$ *hours*

23 MAY 2005 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2, and Module 3.

Each section consists of 1 question.
The maximum mark for each section is 20.
The maximum mark for this examination is 60.
This examination paper consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer this question.

1. Table 1 presents data obtained from a biological investigation that involves two variables x and y.

Table 1

x	20	30	40	50
y	890	1640	2500	3700

It is believed that x and y are related by the formula, $y = bx^n$.

(a) (i) By taking logarithms to base 10 of both sides, convert $y = bx^n$ to the form $Y = nX + d$ where n and d are constants. [4 marks]

   (ii) Hence, express

   a) Y in terms of y

   b) X in terms of x

   c) d in terms of b. [3 marks]

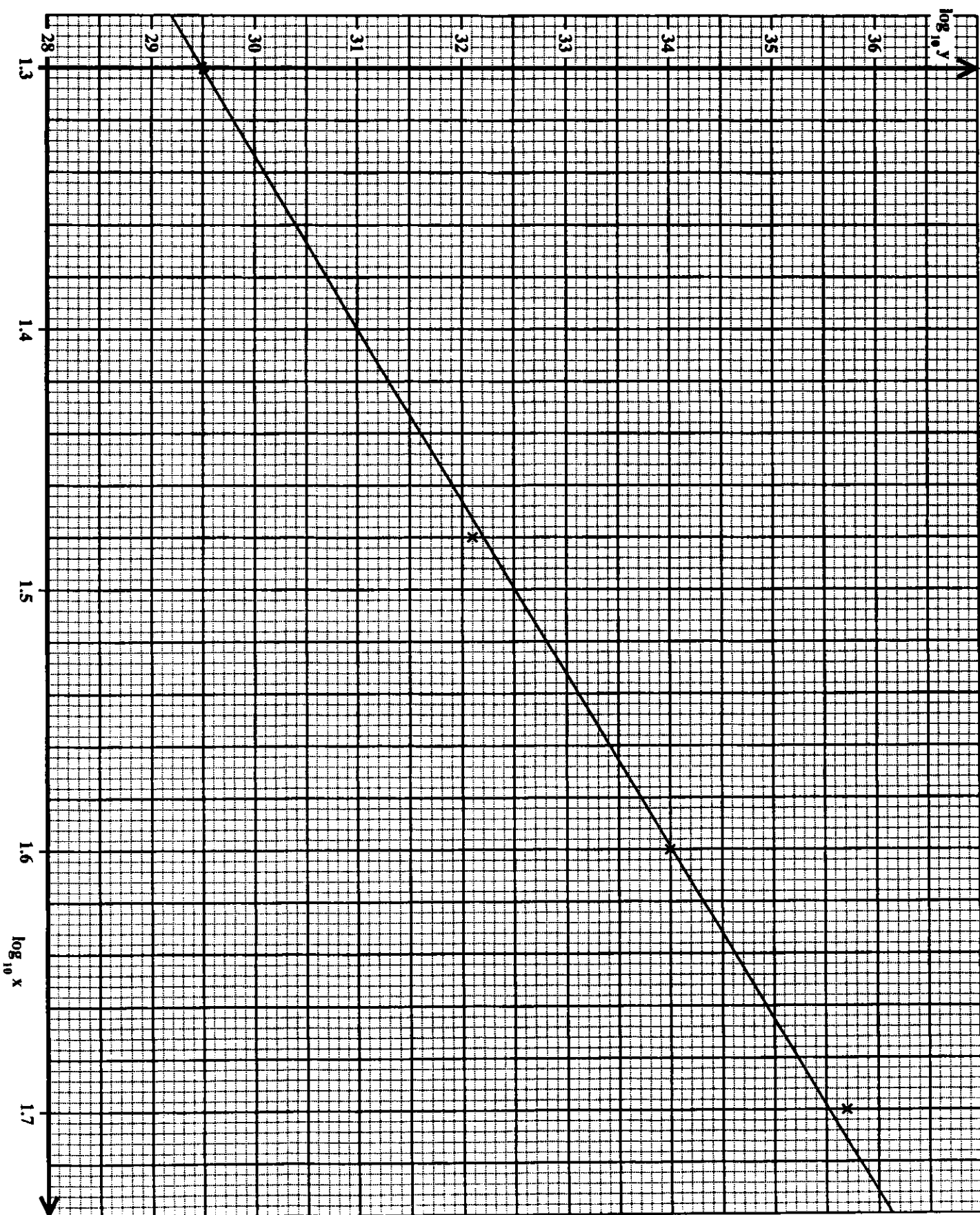(b) Use the data from Table 1 to complete Table 2.

Table 2

$\log_{10}x$	1.30		1.60	
$\log_{10}y$		3.21		3.57

[2 marks]

(c) In the graph on page 3, $\log_{10}x$ is plotted against $\log_{10}y$ for $1.3 \leq x \leq 1.7$.

   (i) Assuming that the 'best straight line' is drawn to fit the data, determine

   a) the gradient of this line [2 marks]

   b) the value of b given that this line passes through (0, 1) [4 marks]

   c) the value of each of the constants, n and d, in Part (a) (i) above. [2 marks]

   (ii) Using the graph, or otherwise, estimate the value of x for which y is 1800. [3 marks]

Total 20 marks

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Section B (Module 2)

Answer this question.

2. Mr John Slick takes out an investment with an investment company which requires making **a fixed payment of $A at the beginning of each year**. At the end of the investment period, John expects to receive a payout sum of money which is equal to the total payments made, together with interest added at the end of EACH year at a rate of r % per annum of the total sum in the fund.

The table below shows information on Mr Slick's investment for the first three years.

Year	Amount at Beginning of Year ($)	Interest ($)	Payout Sum $
1	A	$A \times \frac{r}{100}$	$A + \left(A \times \frac{r}{100} \right)$ $= A \left(1 + \frac{r}{100}\right)$ $= AR$
2	A + AR	$(A + AR) \times \frac{r}{100}$	$(A + AR) + \left[(A + AR) \times \frac{r}{100}\right]$ $= (A + AR)\left(1 + \frac{r}{100}\right)$ $= (A + AR)R$ $= AR + AR^2$
3	$A + AR + AR^2$	$(A + AR + AR^2) \times \frac{r}{100}$	$(A + AR + AR^2)R$ $= AR + AR^2 + AR^3$

(a) Write expressions for

   (i)   the amounts at the beginning of Years 4 and 5   [2 marks]

   (ii) payout sums at the end of Years 4 and 5.   [2 marks]

(b) By using the information in the Table, or otherwise, write an expression for the amount at the beginning of the nth year.   [2 marks]

(c) Show that the payout sum in (b) above is $\dfrac{\$\,AR\,(R^n - 1)}{R - 1}$ for R > 1.   [7 marks]

(d) Find the value of A, to the nearest dollar, when n = 20, r = 5 and the payout sum in (c) above is $500 000.00.   [7 marks]

Total 20 marks

Section C (Module 3)

Answer this question.

The output 3 x 1 matrix Y in a testing process in a chemical plant is related to the input 3 x 1 matrix X by means of the equation Y = AX, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}.$$

(a)   Show that A is non-singular.   [5 marks]

(b)   Show that $X = A^{-1}Y$.   [3 marks]

(c)   Find A^{-1}.   [9 marks]

(d)   Find the input matrix X corresponding to the output matrix $Y = \begin{pmatrix} 19 \\ 34 \\ 42 \end{pmatrix}.$

[3 marks]

Total 20 marks

END OF TEST