

**FORM TP 02234020/SPEC****CARIBBEAN EXAMINATIONS COUNCIL****ADVANCED PROFICIENCY EXAMINATION****PURE MATHEMATICS****UNIT 2****COMPLEX NUMBERS, ANALYSIS AND MATRICES****SPECIMEN PAPER****PAPER 02***2 hours 30 minutes*

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.

The maximum mark for each Module is 50.

The maximum mark for this examination is 150.

This examination consists of 5 printed pages.

**INSTRUCTIONS TO CANDIDATES**

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

**Examination Materials**

Mathematical formulae and tables

Electronic calculator

Ruler and graph paper

## SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) (i) Express the complex number  $\frac{4-2i}{1-3i}$  in the form of  $a + ib$  where  $a$  and  $b$  are real numbers.

[4 marks]

- (ii) Show that the argument of the complex number in (a) (i) above is  $\frac{\pi}{4}$ .

[1 mark]

- (b) (i) Find the complex number  $u = x + iy$ ,  $x, y \in \mathbb{R}$  such that  $u^2 = -5 + 12i$ .

[8 marks]

- (ii) Hence, solve the equation  $z^2 + iz + (1 - 3i) = 0$ .

[6 marks]

- (c) Find the complex number  $z = a + ib$  such that

$$(1 + 3i)z + (4 - 2i)z = 10 + 4i$$

[6 marks]

Total 25 marks

- 2 (a) Find  $e^{3x} \sin 2x \, dx$  [7 marks]

- (b) (i)

- a) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1}(3x)$ . [4 marks]

- b) Hence, find  $\frac{(x+2)}{1+9x^2} dx$ . [4 marks]

- (ii) Show that if  $y = \frac{\ln(5x)}{x^2}$  then  $\frac{dy}{dx} = \frac{1 - \ln(25x^2)}{x^3}$  [5 marks]

- (c) Let  $f(x, y) = x^2 + y^2 - 2xy$ .

- (i) Find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  [2 marks]

- (ii) Show that  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2f(x, y)$  [3 marks]

Total 25 marks

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## SECTION B (MODULE 2)

Answer BOTH questions.

3. (a) (i) Find constants  $A$  and  $B$  such that

$$\frac{1}{(2r-1)(2r+1)} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}. \quad [5 \text{ marks}]$$

- (ii) Hence, find the value of  $S$  where

$$S = \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)}. \quad [5 \text{ marks}]$$

- (iii) Deduce the sum to infinity of  $S$ . [3 marks]

- (b) (i) Find the  $r^{\text{th}}$  term of the series  $1(2) + 2(5) + 3(8) + \dots$  [2 marks]

- (ii) Prove, by Mathematical Induction, that the sum to  $n$  terms of the series in (b) (i) above is  $n^2(n+1)$ . [10 marks]

Total 25 marks

4. (a) Given the series  $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \dots$

- (i) show that the series is geometric [3 marks]

- (ii) find the sum of the series to  $n$  terms. [4 marks]

- (b) Use Maclaurin's Theorem to find the **first** three non-zero terms in the power series expansion of  $\cos 2x$ . [7 marks]

- (c) (i) Expand up to and including the term in  $x^3$

$$\sqrt{\left(\frac{1+x}{1-x}\right)},$$

stating the values of  $x$  for which the expansion is valid. [7 marks]

- (ii) By taking  $x = 0.02$  find an approximation for  $\sqrt{51}$ , correct to 5 decimal places.

[4 marks]

Total 25 marks

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**SECTION C (MODULE 3)****Answer BOTH questions.**

5. (a) Two cards are drawn without replacement from ten cards which are numbered 1 to 10. Find the probability that

- (i) the numbers on **BOTH** cards are even **[4 marks]**
- (ii) the number on one card is odd and the number on the other card is even. **[4marks]**

- (b) A journalist reporting on criminal cases classified 150 criminal cases by the age (in years) of the criminal and by the type of crime committed, violent or non-violent. The information is presented in the table below.

Type of Crime	Age (in years)		
	Less than 20	20 to 39	40 or older
Violent	27	41	14
Non-violent	12	34	22

What is the probability that a case randomly selected by the journalist

- (i) is a violent crime? **[2 marks]**
- (ii) was committed by someone LESS than 40 years old? **[4 marks]**
- (iii) is a violent crime OR was committed by a person LESS than 20 years old? **[3 marks]**

- (c) On a particular weekend, 100 customers made purchases at Green Thumb Garden supply store. Of these 100 customers;

30 purchased tools  
 45 purchased fertilizer  
 50 purchased seeds  
 15 purchased seeds and fertilizer  
 20 purchased seeds and tools  
 15 purchased tools and fertilizer  
 10 purchased tools, seeds and fertilizer.

- (i) Represent the above information on a Venn diagram. **[4 marks]**
- (ii) Determine how many customers purchased:
- a) only tools
- b) seeds and tools but not fertilizer,
- c) tools and fertilizer but not seeds,
- d) neither seeds, tools, nor fertilizer. **[4 marks]**

**Total 25 marks**

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6. (a) Solve for  $x$  the following equation

$$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

**[8 marks]**

- (b) Solve the first order differential equation

$$y \tan x \frac{dy}{dx} = (4 + y^2) \sec^2 x$$

**[4 marks]**

- (c) Given that  $y = u \cos 3x + v \sin 3x$  is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = 30 \sin 3x,$$

find

- (i) the values of the constants  $u$  and  $v$ , **[8 marks]**
- (ii) the general solution of the differential equation. **[5 marks]**

**Total 25 marks**

**END OF TEST**