

FORM TP 2004246



TEST CODE **02134020**

MAY/JUNE 2004

CARIBBEAN EXAMINATIONS COUNCIL
ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 02

$2\frac{1}{2}$ hours

26 MAY 2004 (p.m.)

This examination paper consists of **THREE** sections: Module 1.1, Module 1.2 and Module 1.3.

Each section consists of 2 questions.

The maximum mark for each section is 50.

The maximum mark for this examination is 150.

This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables

Electronic calculator

Graph paper

SECTION A (MODULE 1.1)

Answer BOTH questions.

1. (a) Given that both $(x - 1)$ and $(x - 2)$ are factors of $f(x) = x^3 + mx + n$, find the constants m and n , and the third factor of $f(x)$. [10 marks]

- (b) Find the constants p , q and r such that

$$2y^2 - 9y + 14 \equiv p(y - 1)(y - 2) + q(y - 1) + r. \quad [8 \text{ marks}]$$

- (c) (i) Find the range of values of $x \in \mathbb{R}$ for which

$$|2x - 3| \leq 5.$$

Hence, determine [5 marks]

- (ii) the LEAST possible value of $x + 1$ [1 mark]

- (iii) the GREATEST possible value of $x + 1$. [1 mark]

Total 25 marks

2. (a) (i) Express $f(x) = 12x - 2x^2$ in the form $A + B(x + p)^2$ where A , B and p are real numbers, and find the maximum value of $12x - 2x^2$. [7 marks]

- (ii) Hence, sketch the graph of $f(x) = 12x - 2x^2$, showing clearly its main features. [5 marks]

- (iii) Show that $f(x) = 12x - 2x^2$ is NOT one-to-one. [2 marks]

- (b) (i) a) Copy and complete the following table for the function $f(x) = \sin x$, $0 \leq x \leq 2\pi$.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$f(x)$					

- b) Sketch the graph of f . [4 marks]

- (ii) On a separate diagram, sketch the graph of $f(x) = |\sin x|$, $0 \leq x \leq 2\pi$. [4 marks]

- (iii) By comparing the diagrams in (b)(i) and (ii) above, determine the solution set of the equation $\sin x = |\sin x|$, $0 \leq x \leq 2\pi$. [3 marks]

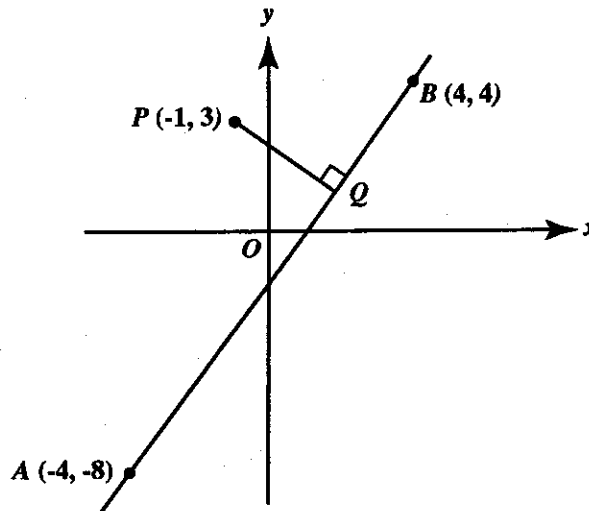
Total 25 marks

GO ON TO THE NEXT PAGE

SECTION B (MODULE 1.2)

Answer BOTH questions.

3. (a) In the diagram below (not drawn to scale), PQ is perpendicular to AQB .



Find

- (i) the equation of the line AB [4 marks]
 - (ii) the equation of the line PQ [4 marks]
 - (iii) the coordinates of the point Q . [4 marks]
- (b) Solve, for $0^\circ \leq \theta \leq 180^\circ$, the equation
- $$6 \cos^2 \theta + \sin \theta = 4. \quad [7 \text{ marks}]$$
- (c) Solve, for $0 \leq x \leq \pi$, the equation
- $$\sin x + \sin 3x = 0. \quad [6 \text{ marks}]$$

Total 25 marks

4. (a) A complex number, z , is expressed in the form $x + iy$, where $x, y \in \mathbf{R}$.

Express the complex number, $w = \frac{z-1}{z+2}$, in a similar form. [8 marks]

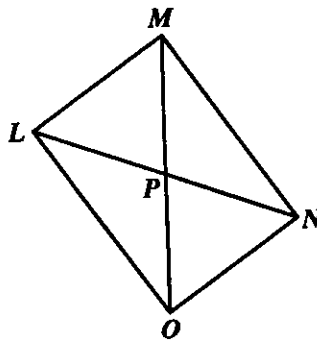
- (b) The argument of w is $\frac{\pi}{4}$.

- (i) Find the equation connecting x and y in the form $ax^2 + by^2 + cx + dy + f = 0$ where a, b, c, d, f are integers. [4 marks]

- (ii) Show that the equation in (i) represents a circle, C . [3 marks]

- (iii) Determine the centre and radius of the circle C . [4 marks]

- (c) The diagram below (not drawn to scale) shows a parallelogram $OLMN$ whose diagonals, OM and LN , intersect at P . The position vectors of L and N relative to the origin, O , are $-3\mathbf{i} + 6\mathbf{j}$ and $2\mathbf{i} + 3\mathbf{j}$ respectively.



Find the position vector of P . [6 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C (MODULE 1.3)

Answer BOTH questions.

5. (a) Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 4x + 3}$. [4 marks]
- (b) Determine the values of $x \in \mathbb{R}$ for which the function $\frac{x+2}{x(x+1)}$ is NOT continuous. [3 marks]
- (c) Given that $y = \frac{x^2 - 1}{x^2 + 1}$,
- (i) find $\frac{dy}{dx}$ in terms of x [5 marks]
- (ii) show that $x(x^2 + 1) \frac{dy}{dx} - 4y = \frac{4}{x^2 + 1}$. [5 marks]
- (d) By investigating the sign of $f'(x)$, determine the range of real values of x for which $x^5 - 5x + 3$ is decreasing [8 marks]

Total 25 marks

6. (a) Find the stationary point(s) of the curve, $f(x) = x^3 - 3x + 2$. [6 marks]
- (b) Determine the nature of the stationary point(s). [3 marks]
- (c) Show that the curve $f(x)$ touches the x -axis at $x = 1$. [4 marks]
- (d) Sketch the curve, $f(x) = x^3 - 3x + 2$, $-2 \leq x \leq 2$. [6 marks]
- (e) Find the area bounded by this curve and the x -axis for $-2 \leq x \leq 1$. [6 marks]

Total 25 marks

END OF TEST