

FORM TP 2011232



TEST CODE **02134032**

MAY/JUNE 2011

CARIBBEAN EXAMINATIONS COUNCIL
ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 03/B

ALGEBRA, GEOMETRY AND CALCULUS

1 ½ hours

08 JUNE 2011 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 3 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2010**

Mathematical instruments

Silent, non-programmable, electronic calculator

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SECTION A (Module 1)

Answer this question.

1. (a) Solve, for x , the equation

$$2^x + 2^{2-x} = 5.$$

[5 marks]

- (b) The functions f and g are defined on \mathbf{R} by

$$f: x \rightarrow 3x + 5 \text{ and } g: x \rightarrow x - 7.$$

- (i) Show that f is one-to-one.

[3 marks]

- (ii) Solve, for x , the equation $f(g(2x + 1)) = f(3x - 2)$.

[4 marks]

- (c) A car manufacturer finds that when x million dollars are spent on research, the profit, $P(x)$, in millions of dollars, is given by

$$P(x) = 15 + 10 \log_4 (x + 4).$$

- (i) What is the expected profit if 12 million dollars are spent on research?

[3 marks]

- (ii) How much money should be spent on research to make a profit of 30 million dollars?

[5 marks]

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) L_1 and L_2 are lines with equations $2x - y = 5$ and $x - 2y = 1$, respectively.

C is a circle with equation $x^2 + y^2 - 12x + 6y + 20 = 0$.

- (i) Show that L_1 and L_2 intersect at a point P on C .

[3 marks]

- (ii) Find the point Q , other than P , at which the line L_1 intersects C .

[4 marks]

- (iii) Find the equation of the tangent to C at P .

[4 marks]

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- (b) (i) Show that $\sin 3A = 3 \sin A - 4 \sin^3 A$. [5 marks]
- (ii) Given the vectors $\mathbf{u} = 2 \sin \theta \mathbf{i} + \cos 2\theta \mathbf{j}$ and $\mathbf{v} = \cos^2 \theta \mathbf{i} + \sin \theta \mathbf{j}$, $0 < \theta \leq \pi$, find the values of θ for which \mathbf{u} and \mathbf{v} are perpendicular. [4 marks]

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) Find $\lim_{x \rightarrow 2} \frac{x^3 - 4x}{x - 2}$. [4 marks]

- (b) (i) Differentiate, with respect to x ,

$$\frac{x}{3x + 4}.$$
 [4 marks]

- (ii) Hence, or otherwise, find

$$\int \frac{16}{(3x + 4)^2} dx.$$
 [3 marks]

- (c) A packaging company wishes to make a **closed** cylindrical container of thin material to hold a volume, V , of 10 cm^3 . The outside surface of the container is $S \text{ cm}^2$, the radius is $r \text{ cm}$ and the height is $h \text{ cm}$.

- (i) Show that $S = 2\pi r^2 + \frac{20}{r}$. [3 marks]

- (ii) Hence, find the **exact** value of r for which S has a **MINIMUM** value.

$$[V = \pi r^2 h, S = 2\pi r^2 + 2\pi rh]$$
 [6 marks]

Total 20 marks

END OF TEST