CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination (CAPE)

PURE MATHEMATICS

Specimen Papers and Solutions and Mark Schemes

Specimen Papers:	Unit 1	-	Paper 01
	Unit 1	-	Paper 02
	Unit 1	-	Paper 03/B
Solutions and Mark Schemes:	Unit 1	-	Paper 01
	Unit 1	-	Paper 02
	Unit 1	-	Paper 03/B

FORM 02134010/SPEC 2007

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS

Paper 01

90 minutes

READ THE FOLLOWING DIRECTIONS CAREFULLY

1. In addition to this test booklet, you should have an answer sheet.

2. Each item in this test has four suggested answers, lettered (A), (B), (C), (D). Read each item you are about to answer, and decide which choice is best.

3. On your answer sheet, find the number which corresponds to your item and blacken the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The expression $(1+\sqrt{3})^2$ is equivalent to

   (A) 4
   (B) 10
   (C) $1+3\sqrt{3}$
   (D) $4+2\sqrt{3}$

   Sample Answer

   Ⓐ Ⓑ Ⓒ ●

   The best answer to this item is "$4+2\sqrt{3}$", so answer space (D) has been blackened.

4. If you want to change your answer, erase your old answer completely and fill in your new choice.

5. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. You can return later to the item omitted.

6. You may do any rough work in the booklet.

7. This test consists of 45 items. You will have 90 minutes to answer them.

8. You may use silent non-programmable calculators to answer questions.

9. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

1. $a(b + c) - b(a + c)$ is equal to

(A) $a(c - b)$
(B) $a(b - c)$
(C) $c(a - b)$
(D) $c(b - a)$

2. Determine the value of x for which $|x + 24| = 7x$ and $x > 0$.

(A) -4
(B) -1
(C) 1
(D) 4

3. The solution set for $|x + 2| < |3x + 2|$ is

(A) $\{x: x > 0\}$
(B) $\{x: 0 < x < 1\}$
(C) $\{x: -1 < x < 0\}$
(D) $\{x: x < -1\} \cup \{x: x > 0\}$

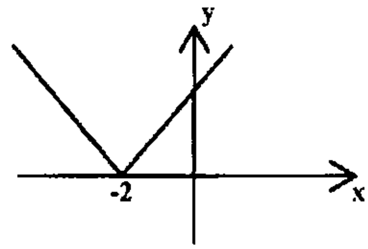
4. If a remainder of 7 is obtained when $x^3 - 3x + k$ is divided by $x - 3$, then k equals

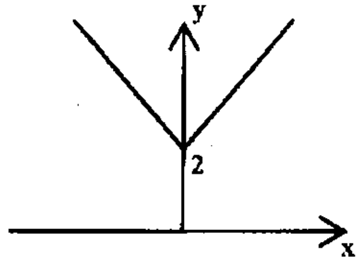
(A) -11
(B) -1
(C) 1
(D) 11

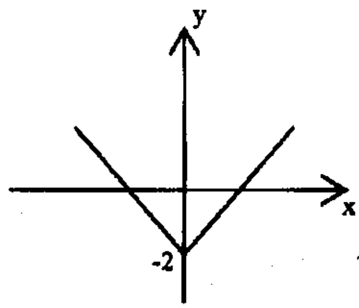
5. If $2x - 1$ is a factor of the polynomial $2x^3 + bx^2 - 8x + 2$, then b is equal to

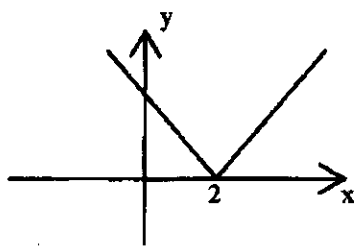
(A) -23
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 7

6. If $f(x) = |x|$, which of the diagrams below represents the graph of $y = f(x) + 2$?

(A)


(B)


(C)


(D)


7. $\dfrac{4(x^{16})^{\frac{1}{2}}}{x^2(x^{-3})}$ simplifies to

(A) $2x^7$
(B) $2x^9$
(C) $4x^7$
(D) $4x^9$

8. If $3^{2x+1} - 4(3^x) + 1 = 0$ then which of the statements below is true?

I. $x = -1$
II. $x = 1$
III. $x = 0$
IV. $x = 2$

(A) I or II only
(B) II or III only
(C) I or III only
(D) II or IV only

9. $\dfrac{2\sqrt{3}+3\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ can be simplified correctly to

(A) $\sqrt{6}$

(B) $2\sqrt{5}$

(C) $12+5\sqrt{6}$

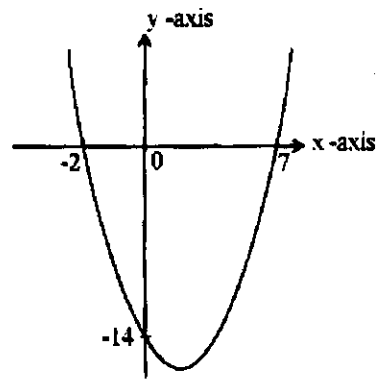
(D) $\dfrac{12+5\sqrt{6}}{5}$

10. $2x^2 + 12x - 11 =$

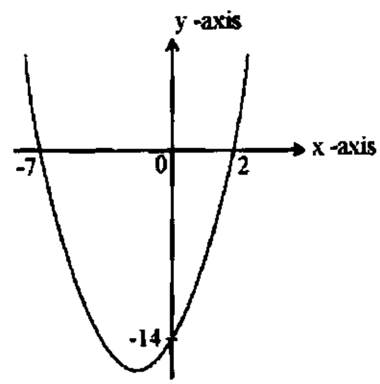
(A) $2(x+3)^2 - 20$
(B) $2(x+6)^2 - 23$
(C) $2(x+3)^2 - 29$
(D) $2(x+3)^2 - 11$

11. Which one of the graphs below **best** represents the equation $y = x^2 - 5x - 14$?

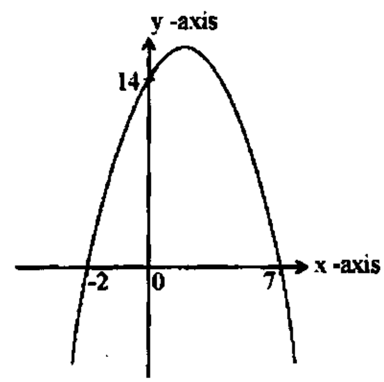
(A)



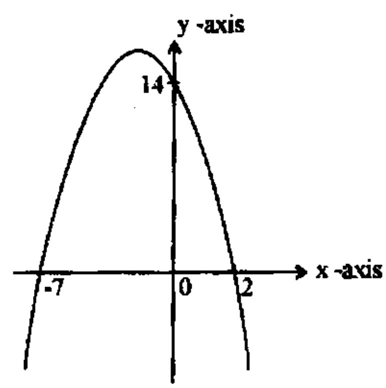
(B)



(C)



(D)



GO ON TO THE NEXT PAGE

12. The roots of the quadratic equation $2x^2 - 4x + 1 = 0$ can be described **completely** as

(A) real
(B) not real
(C) real and equal
(D) real and distinct

13. If α and β are the roots of the equation $3x^2 + 6x - 1 = 0$, then $2\alpha^2\beta^2$ equals

(A) $\frac{2}{9}$

(B) $\frac{4}{9}$

(C) $\frac{2}{3}$

(D) $\frac{4}{3}$

14. The roots of $x^3 - 2x^2 - x + 2 = 0$ are

I $x = 1$
II $x = -1$
III $x = 2$
IV $x = -2$

(A) I and II only
(B) I, II and III only
(C) II and III only
(D) II, III and IV only

15. The real values of x for which $\frac{x+1}{x-2} < 0$ are given by

(A) $x > -1$
(B) $x < 2$
(C) $-1 < x < 2$
(D) $x < -1$ or $x > 2$

16. The value that θ, $0 \leq \theta \leq \pi$, which satisfies the equation $2\cos^2\theta + 3\cos\theta - 2 = 0$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

17. Given that α is an **acute angle** and $\tan\alpha = \frac{3}{4}$ then $\sin(\frac{\pi}{2} - \alpha)$ equals

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{3}{4}$

(D) $\frac{4}{5}$

18. $\cos(A - B) - \cos(A + B) =$ equals

(A) $2\sin A \sin B$
(B) $-2\sin A \cos B$
(C) $2\cos A \sin B$
(D) $2\cos A \cos B$

GO ON TO THE NEXT PAGE

19. The solution of the equation $2 \sin x = \sqrt{2}$, where $\pi < x < \frac{3\pi}{2}$, *is*

(A) $\frac{-5\pi}{4}$

(B) $\frac{-3\pi}{4}$

(C) $\frac{\pi}{4}$

(D) $\frac{5\pi}{4}$

20. Given that
$5 \cos\theta - 12\sin\theta = 13\cos(\theta + 67.4)$,
which of the following equations has solutions for all values of θ?

(I) $5 \cos \theta$-12 sin $\theta = 6$
(II) $5 \cos \theta$-12 sin $\theta = -10$
(III) $5 \cos \theta$-12 sin $\theta = 17$
(IV) $5 \cos \theta$-12 sin $\theta = -13$

(A) I only
(B) I and II only
(C) II, III and IV only
(D) I, II and IV only

21. The coordinates of the points A and B are (2, -3) and (-10, -5) respectively. The gradient of the line perpendicular to the segment AB is

(A) +6
(B) -6
(C) $\frac{2}{3}$
(D) -1

22. The lines 2y-3x -13 = 0 and y + x + 1 = 0 intersect at the point where

(A) x = -3, y = 2
(B) x = 3, y = 2
(C) x = -3, y = -2
(D) x = -7, y = 10

23. The centre of the circle
$(x-3)^2 + (y+2)^2 = 25$ is

(A) (-2, 3)
(B) (2, -3)
(C) (-3, 2)
(D) (3, -2)

24. The curve with parametric representation $x = a \cos$, $y = b\sin\theta$ has Cartesian equation

(A) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

(B) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$

(C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

25. The line $y = x - 3$ intersects the circle $(x-2)^2 + (y+3)^2 = 10$ at the points

(I) (3, 0)
(II) (-3, 0)
(III) (1, -4)
(IV) (-1, -4)

(A) I and II only
(B) II and III only
(C) I and IV only
(D) I, III and IV only

GO ON TO THE NEXT PAGE

26. The points of intersection of the curves $y^2 = 4ax$ and $x^2 = 4ay, a \in R, a > 0$ are

(I) (0, 4a)
(II) (0, 0)
(III) (4a, 4a)
(IV) (4a, 0)

(A) I and II only
(B) II and III only
(C) I and III only
(D) I, II and IV only

27. The Cartesian equation of the curve with parametric representation $x = t^2 - 2,\ y = 5t^4 - 3$ is

(A) $y = 5(x+2)^2 - 3$
(B) $y = 5(x-2)^2 - 3$
(C) $y = 5(x+2)^2 + 3$
(D) $y = (x-2)^2 - 3$

28. If the length of the vector

$\mathbf{x} = 5\mathbf{i} - (k-2)\mathbf{j}$ is $\sqrt{34}$ and k is real, then

(I) k = 5
(II) k = -5
(III) k = 1
(IV) k = -1

(A) I or II only
(B) I or IV only
(C) II or III only
(D) I, III or IV only

29. The value of the real number t for which the two vectors

$\mathbf{p} = t\mathbf{i} + 3\mathbf{j}$ and $\mathbf{q} = -9\mathbf{i} + 7\mathbf{j}$

are perpendicular is

(A) $\frac{3}{7}$
(B) $\frac{7}{3}$
(C) 7
(D) 21

30. The value of the real number k for which the two vectors

$\mathbf{a} = 4\mathbf{i} + k\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j}$

are parallel is

(A) -6
(B) $\frac{-3}{4}$
(C) $\frac{4}{3}$
(D) 6

31. As x approaches 3, the limit of $\frac{2x^2 - 18}{x - 3}$ is

(A) 0
(B) 6
(C) 12
(D) ∞

32. As x approaches zero, the limit of $\frac{\sin\ x}{4x}$ is

(A) 0
(B) $\frac{1}{4}$
(C) $\frac{x}{4}$
(D) ∞

GO ON TO THE NEXT PAGE

33. Given $f(x)=(x-2)(x^3+5)$, $f'(x)$ is

(A) $3x^2$

(B) $-2(3x^2+5)$

(C) $4x^3-6x^2+5$

(D) $x^4-2x^3+5x-10$

34. Given that $f(x)=\frac{2}{x^2}$, then $f'(x)$ equals

(A) $\frac{-4}{x^3}$

(B) $\frac{6}{x^2}$

(C) $\frac{-1}{x}$

(D) $\frac{-2}{x}$

35. If $f(x)=(x^2+1)^2$, then $f'(2)$ equals

(A) 5
(B) 10
(C) 25
(D) 40

36. If $y=\cos x-\sin x$, then $\frac{d^2y}{dx^2}$ is

(A) $\cos x+\sin x$
(B) $\cos x-\sin x$
(C) $-\cos x+\sin x$
(D) $-\cos x-\sin x$

37. At $x=2$, the function $2x^3-6x^2+5$

(A) is decreasing
(B) is increasing
(C) has a minimum value
(D) has a maximum value

38. $\frac{d}{dx}(x^2 \sin x)$ is

(A) $2x \sin x$
(B) $x^2 \cos x$
(C) $2x \sin x - x^2 \cos x$
(D) $2x \sin x + x^2 \cos x$

39. The real values of x for which the function $f(x)=x^3-5x^2-3$ is increasing are

(I) $x<0$

(II) $x>\frac{10}{3}$

(III) $x>0$

(IV) $x<\frac{10}{3}$

(A) I or II only
(B) II or III only
(C) I or IV only
(D) I, II, III or IV

40. The x co-ordinates of the stationary points on the curve $y=4x^3-3x$ are

(A) -2 and 2

(B) $-\frac{1}{2}$ and $\frac{1}{2}$

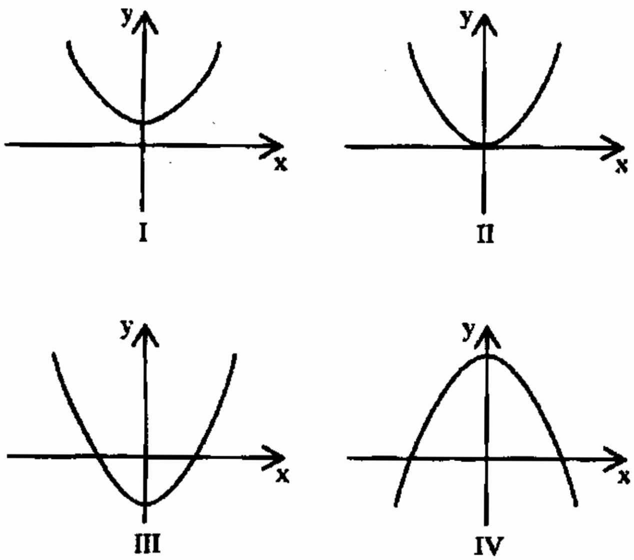
(C) 0 and $\frac{1}{2}$

(D) $\frac{1}{4}$ and 3

41. The equation of the normal at (1, -3) to the curve $y = x^3 - 8x + 4$ may be expressed as

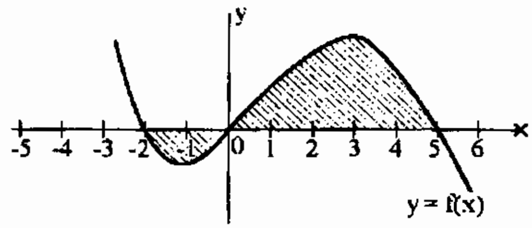
(A) $x - 5y - 16 = 0$
(B) $x - 5y - 14 = 0$
(C) $5x - y - 8 = 0$
(D) $5x + y - 2 = 0$

42. Which of the following graphs belong to the family of curves with the equation $y = \int 2x\,dx$?



(A) II only
(B) I and III only
(C) I, II and III only
(D) II, III and IV only

43. The total shaded area in the diagram below is given by



(A) $\int_{-2}^{5} f(x)\,dx$

(B) $-\int_{-2}^{0} f(x)dx + \int_{0}^{5} f(x)dx$

(C) $\int_{-5}^{-2} f(x)\,dx$

(D) $\int_{-2}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx$

44. The volume (in units^3) generated when the region bounded by the graphs of $y^2 = x + 3$, $x = 0$ and $x = 3$ is rotated through 2π radians about the x - axis is

(A) $\frac{27}{2}$

(B) 63

(C) $\frac{27\pi}{2}$

(D) 63π

45. The rate of **decay** of a radioactive substance is **directly proportional** to the amount, x, of the substance remaining after time t. A model for this situation is given by

(A) $-\frac{dx}{dt} = \frac{k}{x}$

(B) $-\frac{dx}{dt} = kx$

(C) $\frac{dx}{dt} = kx^2$

(D) $\frac{dx}{dt} = kx$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

FORM TP 02134020/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1

ALGERBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 02

2 hours 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 6 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) (i) Construct a table for the function, $f(x) = x^3 - 3x + 2$ for $x = 0, 0.5, 1.0, 1.5, 2.0$. **[2 marks]**

(ii) Using a scale of 5 cm to represent 1 unit on the domain and 2 cm to represent 1 unit on the co-domain, draw the graph of $f(x), 0 \le x \le 2$. **[3 marks]**

(iii) On the same axes, draw the graph of $g(x) = x - 1$ for $0 \le x \le 2$. **[1 mark]**

(iv) Estimate to 1 decimal place

a) the value(s) of x for which $f(x) = g(x)$ **[1 mark]**

b) the range of values of x for which $f(x) < g(x)$. **[1 mark]**

(v) Use the information from your graph in (ii) above to obtain a linear factor of $f(x)$. **[2 marks]**

(vi) Hence, or otherwise, factorise completely $x^3 - 3x + 2$. **[5 marks]**

(b) The roots of the quadratic equation $x^2 - 3x - 1 = 0$ are α and β.

Without solving the equation

(i) state the values of $\alpha + \beta$ and $\alpha\beta$ **[2 marks]**

(ii) find the value of $\alpha^2 + \beta^2$ **[2 marks]**

(iii) obtain the equation whose roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$. **[6 marks]**

Total 25 marks

2 (a) Two of the roots of the cubic equation

$2x^3 + px^2 + qx + 2$ are -1 and $\frac{1}{2}$.

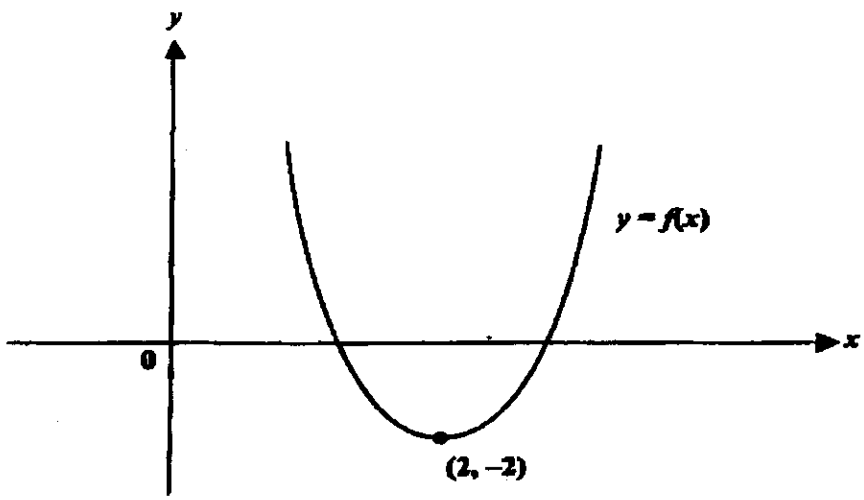
Find

(i) the values of the constants p and q **[4 marks]**

(ii) the remaining root of the equation. **[2 marks]**

(b) Prove by Mathematical Induction that $\sum_{r=1}^{n}(6r+5) = n(3n+8)$. **[10 marks]**

(c) The diagram below, **not drawn to scale,** shows the graph of $y = f(x)$ which has a minimum at (2,-2).



Copy this diagram and on the **same axes** sketch the graphs of

(i) $y = f(x-1)$ **[3 marks]**

(ii) $y = f(x)+3$ **[3 marks]**

(iii) $y = |f(x)|$. **[3 marks]**

Total 25 marks

GO TO NEXT PAGE

SECTION B (MODULE 2)

Answer BOTH questions.

3\. (a) (i) Prove the identity $\dfrac{\sin 3\theta + \sin\theta}{\cos 3\theta + \cos\theta} = \tan 2\theta$. **[4 marks]**

(ii) Solve the equation: $\sin\theta + \sin 2\theta + \sin 3\theta = 0, 0 \leq \theta \leq 2\pi$.

[7 marks]

(b) (i) Express $f(\theta) = 8\cos\theta + 6\sin\theta$ in the form $r\cos(\theta - \alpha)$ where $r > 0$, $0 < \alpha < 90$. **[3 marks]**

(ii) Determine the minimum value of

$$g(\theta) = \frac{10}{10 + 8\cos\theta + 6\sin\theta} \quad \text{and}$$

state the value of θ for which $g(\theta)$ is at its minimum. **[4 marks]**

(c) (i) The position vectors of two points, P and Q, relative to a fixed origin O are $3t^2\mathbf{i} + 2\mathbf{j}$ and $4\mathbf{i} - 2t\mathbf{j}$ respectively where $t > 0$. Find the value of t such that $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are perpendicular. **[4 marks]**

(ii) The points A and B are represented by the vectors $\mathbf{a} = 7\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + 5\mathbf{j}$ relative to a fixed origin O. Find the unit vector $\overrightarrow{AB}$.

[3 marks]
Total 25 marks

4 (a) (i) Show that $x + 2y = 7$ is a tangent to the circle $x^2 + y^2 - 4x - 1 = 0$.

[8 marks]

(ii) Determine the equation of the tangent diametrically opposite to the tangent $x + 2y = 7$ of the curve $x^2 - 4x + y^2 = 1$. **[4 marks]**

(b) The parametric equations of a curve, C, are given by

$$x = \frac{t}{1+t} \quad \text{and} \quad y = \frac{t^2}{1+t}.$$

Determine the Cartesian equation of C. **[5 marks]**

(c) (i) The line L passes through the points A (-2, 0) and B (0, 1). Find the equation of L. **[3 marks]**

(ii) The line M is perpendicular to L and passes through the point $\left(\frac{3}{2}, 1\frac{1}{2}\right)$. Find the equation of M. **[2 marks]**

(iii) Determine the point of intersection of L and M. **[3 marks]**
Total 25 marks

GO TO NEXT PAGE

SECTION C (MODULE 3)

Answer BOTH questions.

5. (a) Show that $\lim_{h \to 0} \frac{h}{\sqrt{x+h} - \sqrt{x}} = 2\sqrt{x}$. **[6 marks]**

(b) The function $f(x)$ is such that $f'(x) = 9x^2 + 4x + c$, where c is a constant. Given that $f(2) = 14$ and $f(3) = 74$. Find the value of $f(4)$. **[7 marks]**

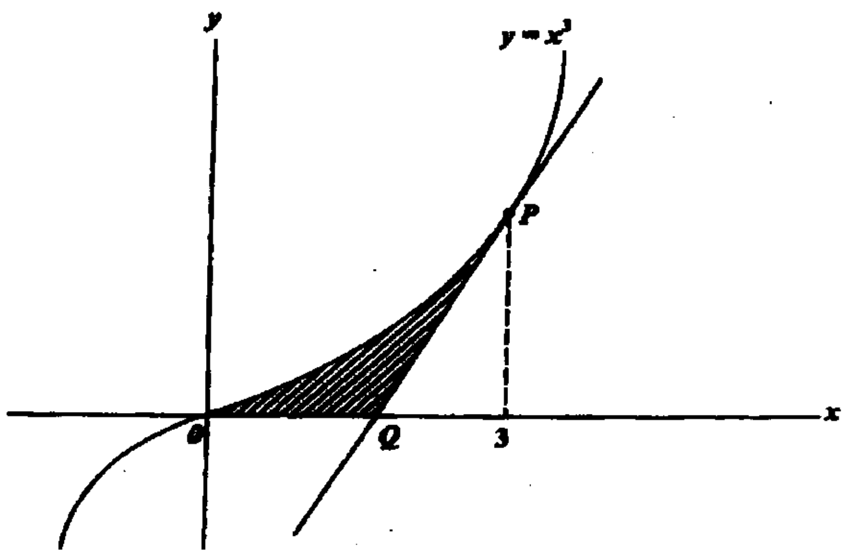
(c) If $y = \frac{x}{1 + x^2}$, show that

(i) $\frac{dy}{dx} = \frac{1}{(1+x^2)^2} - y^2$ **[6 marks]**

(ii) $\frac{d^2y}{dx^2} = \frac{2y(x^2 - 3)}{(1+x^2)^2}$ **[6 marks]**

Total 25 marks

6 (a) The diagram below, **not drawn to scale** is a sketch of the curve $y = x^3$ and the tangent PQ to the curve at $P(3, 27)$.



(i) Find

a) the equation of the tangent PQ **[4 marks]**

b) the coordinates of Q. **[1 mark]**

GO TO NEXT PAGE

(ii) Calculate

a) the area of the shaded region in the diagram **[5 marks]**

b) the volume of the solid generated when the shaded region is rotated completely about the x-axis, giving your answer in terms of π. **[5 marks]**

[If necessary, the volume V of a cone is given by $V = \frac{1}{3}\pi r^2 h$.]

(b) The gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 8x + 5.$$

The curves passes through the point (0, 3).

(i) Find the equation of this curve. **[3 marks]**

(ii) Find the coordinates of the two stationary points of the curve in (b) (i) above and identify the nature of **each.** **[7 marks]**

Total 25 marks

END OF TEST

FORM TP 02134032/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS
PAPER 03/B

1 hour 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

SECTION A (MODULE 1)

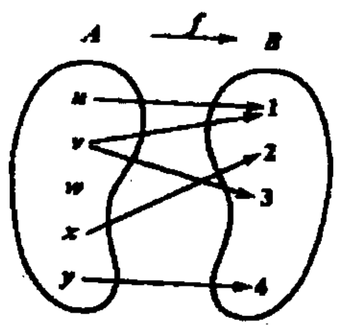
Answer this question.

1. (a) Solve, for x, the equations

(i) $\dfrac{3^{x^2}}{81} = 9^{x+2}$ [7 marks]

(ii) $|x+4| = |2x-1|$. [7 marks]

(b) A coach of an athletic club has five athletes, u, v, w, x and y, in his training camp. He makes an assignment, f, of athletes u, v, x and y to physical activities 1, 2, 3 and 4 according to the diagram below in which A= {u, v, w, x, y}, B = {1, 2, 3, 4} and $f = \{(u, 1), (v, 1), (v, 3), (x, 2), (y, 4)\}$.



(i) State ONE reason why the assignment f from A to B is **not** a function. [1 mark]

(ii) State TWO changes that the coach would need to make so that the assignment f, becomes a function $g: A \rightarrow B$. [2 marks]

(iii) Express the function $g: A \rightarrow B$ in (ii) above as a set of ordered pairs. [3 marks]

Total 20 marks

SECTION B (MODULE 2)

Answer this question.

2. (a) In an experiment, the live weight, w grams, of a hen was found to be a linear function, f, of the number of days, d, after the hen was placed on a special diet, where $0 \leq d \leq 50$. At the beginning of the experiment, the hen weighed 500 grams and 25 days later it weighted 1 500 grams.

(i) Copy and complete the table below.

***d* (days)**		25
***w* (gms)**	500	

[1 mark]

(ii) Determine

a) the linear function, f, such that $f(d) = w$ [3 marks]

b) the expected weight of a hen 10 days after the diet began. [2 marks]

(iii) After how may days is the hen expected to weigh 2 180 grams? [2 marks]

(b) (i) Show that $(\tan\theta - \sec\theta)^2 \equiv \dfrac{\sin^2\theta - 2\sin\theta + 1}{\cos^2\theta}$. [3 marks]

(ii) Hence, show that $\dfrac{1 - \sin\theta}{1 + \sin\theta} \equiv (\tan\theta - \sec\theta)^2$. [4 marks]

(c) The position vectors of points A and C relative to the origin, O, are $4\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + 7\mathbf{j}$ respectively.

Find

(i) $\cos A\hat{O}C$ [4 marks]
(ii) $A\hat{O}C$. [1 mark]

Total 20 marks

SECTION C (MODULE 3)

Answer this question.

3. (a) (i) By expressing $x-4$ as $(\sqrt{x}+2)\,(\sqrt{x}-2)$, find $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4}$.

[3 marks]

(ii) Hence, find $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x^2-5x+4}$. [3 marks]

(b) If $y = \dfrac{A}{x} + Bx$, where A and B are constants, show at $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = y$.

[5 marks]

(c) An observant mathematician, living beside a river, notes that, when the river begins to flood after rain, the level rises at the rate of $(14 - 3t)$ inches per hour, t being the number of hours that have elapsed since the flooding started.

(i) How many hours after the flooding started will the level reach its highest point? [5 marks]

(ii) By how many inches will the level have risen at the highest point?

[1 mark]

(iii) After how many hours will the level reach a step which is 2ft 6 in. above the level of the river? [3 marks]

Total 20 marks

END OF TEST

02134010/CAPE/K - 2007

CARIBBEAN EXAMINATIONS COUNCIL
HEADQUARTERS

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 1

PAPER 01

KEY

CARIBBEAN EXAMINATIONS COUNCIL

Pure Mathematics Unit 1

Item	Key	Item	Key
1	C	24	C
2	D	25	C
3	D	26	B
4	A	27	A
5	D	28	B
6	B	29	B
7	D	30	A
8	C	31	C
9	A	32	B
10	C	33	C
11	A	34	A
12	D	35	D
13	A	36	C
14	B	37	C
15	C	38	D
16	C	39	A
17	D	40	B
18	A	41	A
19	D	42	C
20	D	43	B
21	B	44	C
22	A	45	B
23	D		

02134020/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER
PAPER 02

SOLUTIONS
&
MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a) (i) $f(x) = x^3 - 3x + 2$

x	0	0.5	1.0	1.5	2.0
x^3	0	0.13	1.0	3.38	8.0
$-3x + 2$	2	0.5	-1.0	-2.5	-4.0
$f(x)$	2	0.63	0	0.88	4.0

(2 marks)

[2 marks]

(ii) Use of correct scales (1 mark)

Labelled axes and curve/ lines (1 mark)

Smooth curve (1 mark)

[3 marks]

(iii) Labelled line – g (x) (1 mark) [1 mark]

(iv) a) x = 1.0, 1.2 (1 mark) [1 mark]

b) $1.0 \leq x \leq 1.2$ (1 mark) [1 mark]

(v) From graph (1,0) lies on f(x) (1 mark)

When x = 1, f(x) = 0

$\therefore$ x -1 is a factor of f(x). (1 mark)

[2 marks]

(vi) To factorise $x^3 - 3x + 2$

Consider x = 1, $1^3 - 3 + 2 = 0$ or from graph

$x - 1$ is a factor of $x^3 - 3x + 2$

By long division

$$
\begin{array}{r}
x^2 + x - 2 \\
x - 1 \overline{)\, x^3 - 3x + 2} \\
\underline{-(x^3 - x^2)} \\
x^2 - 3x \\
\underline{-(x^2 - x)} \\
-2x + 2 \\
\underline{-(-2x + 2)}
\end{array}
$$

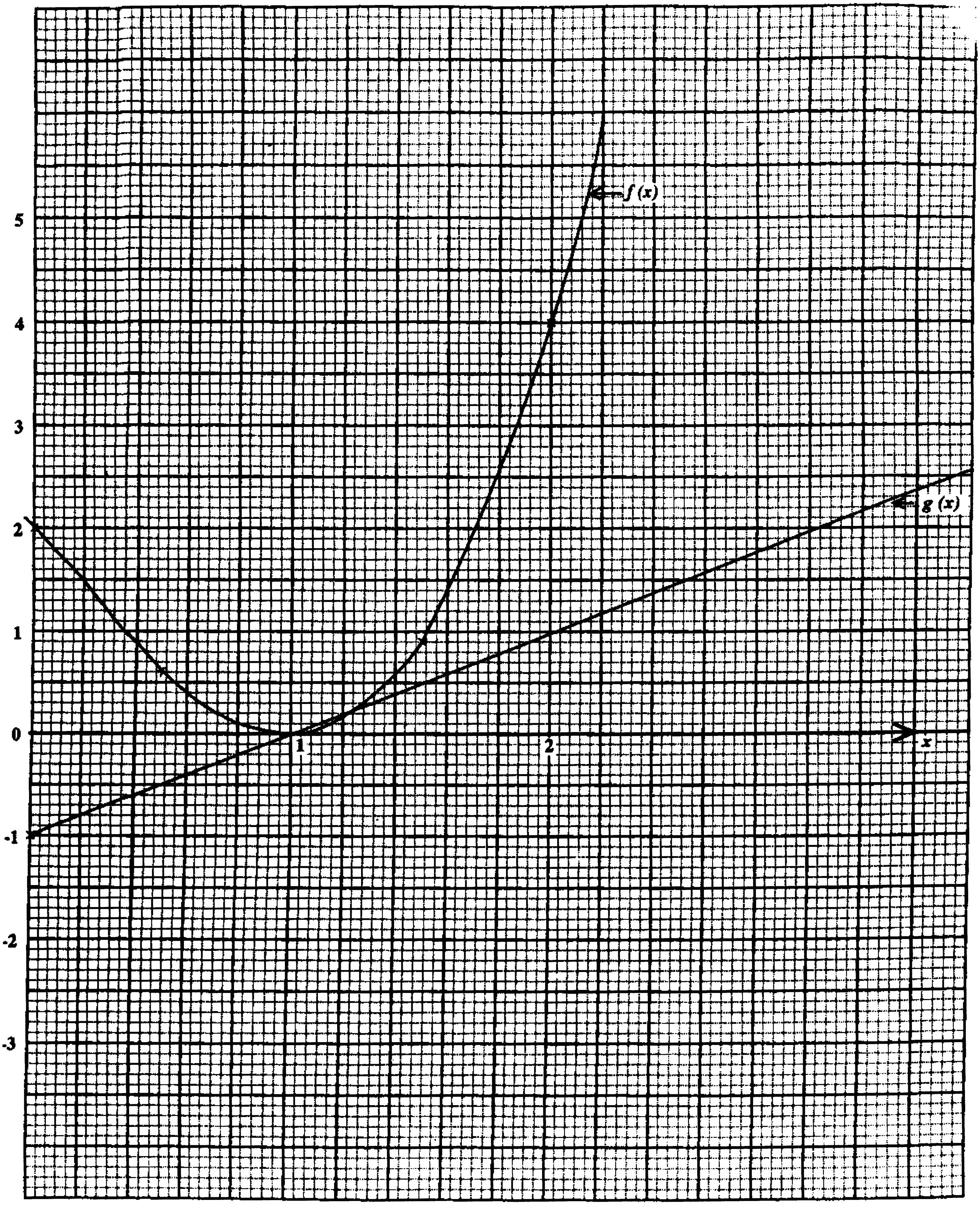
(3 marks)

The complete factors are

$(x - 1)\ (x - 1)\ (x + 2)$. (2 marks)

[5 marks]

(a) (ii)



(b) Since the roots of $x^2 - 3x - 1 = 0$ are α and β

then

(i) $\alpha + \beta = 3$ and $\alpha\beta = -1$ (2 marks)

[2 marks]

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ (1 mark)

$= 9 + 2$

$= 11$ (1 mark)

[2 marks]

(iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\beta + 2\alpha}{\alpha\beta} = \frac{2(\alpha + \beta)}{\alpha\beta} = -6$ (3 marks)

and $\frac{2}{\alpha}\frac{2}{\beta} = \frac{4}{\alpha\beta} = -4$ (2 marks)

The equation, whose roots sum to – 6 and have a product – 4, is given by

$x^2 + 6x - 4 = 0$ (1 mark)

[6 marks]

Total 25 marks

Specific Objectives: (d) 3, 5, 8; (f) 5 (i)

Question 2

(a) Let $f(x) = 2x^3 + px^2 + qx + 2$

(i) $f(-1) = 0 \Rightarrow -2 + p - q + 2 = 0 \Rightarrow p = q$ (1 mark)

$f\left(\frac{1}{2}\right) = 0 \Rightarrow \frac{1}{4} + \frac{p}{4} + \frac{q}{2} + 2 = 0 \Rightarrow p + 2q = -9$ ([2 mark)

$\Rightarrow p = q = -3$ (1 mark)

(ii) $f(x) = (2x-1)(x+1)(x-k)$ (1 mark)

$\equiv 2x^3 + px^2\ qx + 2$

$\Rightarrow k = 2$

$\Rightarrow$ the remaining root is 2 (1 mark)

(b) Let p_n be the statement $\sum_{r=1}^{n}(6r + 5) = n(3n + 8)$

For n = 1, L.H.S. p_n, is 6 + 5 = 11 and R.H.S. of p_n

= 1(3 + 8)

= 11 (1 mark)

So p_n is true for n = 1 (1 mark)

Assume that p_n is true for $n = k$, i.e

$11 + 17 + \ldots.. + (6k + 5) = k(3k + 8)$ ....I (1 mark)

Then, we need to prove p_n is true for $n = k + 1$, i.e from I above

From I,

$11 + 17 + \ldots + (6k + 5) + [6(k + 1) + 5]$

$= k(3k + 8) + [6(k + 1) + 5]$ (2 marks)

$= 3k^2 + 8k + 6k + 6 + 5$

$= 3k^2 + 14k + 11$

$= (3k + 11)(k + 1)$ (2 marks)

$\therefore\ 11 + \ldots + 6k + 5 + [6(k + 1) + 5] = (k + 1)[(k + 1) + (k + 1) + (k + 1) + 8]$

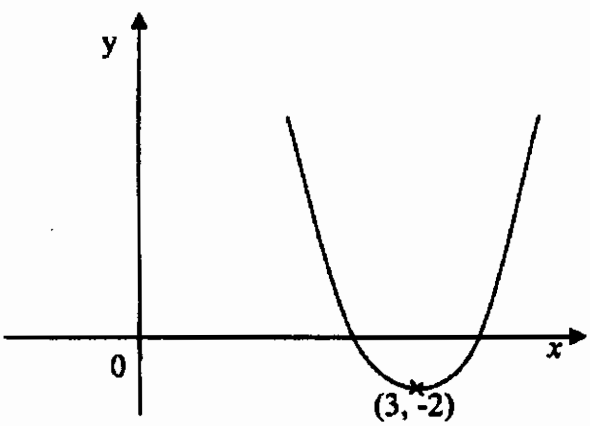
$= (k + 1)[3(k + 1) + 8]$ (1 mark)

Thus, if p_n is true when $n = k$, it is also true with $n = (k + 1)$; (1 mark)

i.e. $\sum_{r=1}^{n}(6r + 5) = n(3n + 8)\ \forall n \in N$ (1 mark)

[10 marks]

(c) (i)



$y = f(x - 1)$

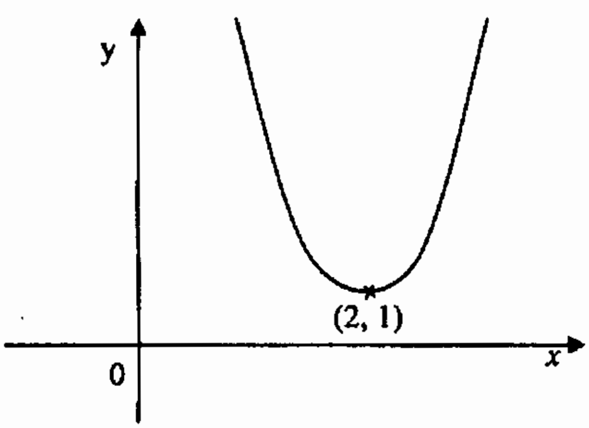
1 transformation

1 point

1 sketch

[3 marks]

(ii)



$y = f(x) + 3$

1 transformation

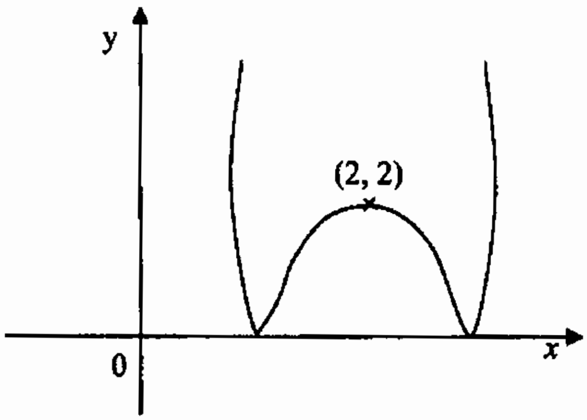
1 point

1 sketch

(3 marks)

[3marks]

(iii)



$y = |f(x)|$

1 transformation

1 point

1 sketch

(3 marks)

[3 marks]

Total 25 marks

Specific Objectives: (a) 7, 8; (b) 3, 4, 6; (d)10

SECTION B
(MODULE 2)

Question 3

(a) (i) $\text{LHS} \equiv \dfrac{\sin 3\theta + \sin\theta}{\cos 3\theta + \cos\theta}$ (1 mark)

$$\equiv \frac{\sin\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{(3\theta-\theta)}{2}\right)}{\cos\left(\frac{3\theta+\theta)}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)}$$ (1 mark)

$$\equiv \frac{\sin 2\theta}{\cos 2\theta}$$ (1 mark)

$\equiv \tan 2\theta$ (1 mark)

$\equiv \text{RHS}$ [4 marks]

(ii) $2\sin\left(\dfrac{3\theta+\theta}{2}\right)\cos\left(\dfrac{3\theta-\theta}{2}\right) + \sin 2\theta = 0$ (1 mark)

$\Rightarrow 2\sin 2\theta\ \cos\theta + \sin 2\theta = 0$ (1 mark)

$\Rightarrow \sin 2\theta\,(2\cos\theta + 1) = 0$ (1 mark)

$\Rightarrow \sin 2\theta = 0$, that is, $\theta = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$ and (3 marks)

$\cos\theta = \dfrac{-1}{2}$, that is, $\theta = \dfrac{2\pi}{3}$, $\dfrac{4\pi}{3}$ (1 mark)

[7 marks]

(b) (i) $r = \sqrt{(6^2 + 8^2)} = 10,\ x = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$ (2 marks)

$\therefore\ f(\theta) = 10\cos(\theta - 36.9^\circ)$ (1 mark)

[3 marks]

(ii) $g(\theta) = \dfrac{10}{1\theta + 10\cos(\theta - 36.9^\circ)}$ (1 mark)

Minimum value of $g(\theta)$ occurs when denominator has maximum value i.e.

$\cos(\theta - 36.9) = 1.$ (1 mark)

Min $g(\theta) = \dfrac{10}{10+10} = \dfrac{1}{2}$ occurs (when $\theta - 36.9 = 0$), when $\theta = 36.9.$ (2 marks)

[4 marks]

(c) (i) $(3t^2\mathbf{i} + 2\mathbf{j})(4\mathbf{i} - 2t\mathbf{j}) = 0$ (1 mark)

$12t^2 - 4t = 0$ (1 mark)

$4t(3t - 1) = 0,\ t > 0$ (1 mark)

$t = \frac{1}{3}$ (1 mark)

[4 marks]

(i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \mathbf{b} - \mathbf{a}$

$\overrightarrow{AB} = (2\mathbf{i} + 5\mathbf{j}) - (7\mathbf{i} - 2\mathbf{j})$ (1 mark)

$= -5\mathbf{i} + 7\mathbf{j}$ (1 mark)

$\hat{\overrightarrow{AB}} = \dfrac{1}{\sqrt{(5^2 + 7^2)}}(-5\mathbf{i} + 7\mathbf{j})$ (1 mark)

$= \dfrac{1}{\sqrt{(74)}}(-5\mathbf{i} + 7\mathbf{j})$ [3 marks]

Total 25 marks

Specific objectives: (a) 4, 5, 9,12,13,14; (c) 3, 4, 5, 6, 7, 8, 9

Question 4

(a) (i) $x + 2y = 7$ is a tangent to $x^2 + y^2 - 4x - 1 = 0$ if $x + 2y = 7$ touches the circle at one point. (1 mark)

Now $x = 7 - 2y$ (1 mark)

$(7y - 2y)^2 - 4(7y - 2y) + y^2 - 1 = 0$ (1 mark)

$\Rightarrow\ y^2 - 4 + 4 = 0$

$\Rightarrow\ (y - 2)^2 = 0$ (2 marks)

$\Rightarrow\ y = 2$ (1 mark)

$\Rightarrow$ when $y = 2$, $x = 3$ (1 mark)

So line touches curve once at (3, 2) (1 mark)

[8 marks]

(ii) Let Q $\equiv$ point diametrically opposite to (3, 2).

$\frac{3+x}{2} = 2,\ x = 1$ (1 mark)

$\frac{2+y}{2} = 0,\ y = -2$ (1 mark)

$\therefore Q = (1, -2)$

Tangent through Q: $y + 2 = \frac{-1}{2}(x-1)$

$2y + x + 3 = 0$ (2 marks)

[4 marks]

(b) $x(1+t) = t \qquad y(1+t) = t^2$

$\Rightarrow \frac{y(1+t)}{x(1+t)} = \frac{t^2}{t}$ (1 mark)

$\Rightarrow \frac{y}{x} = t$ (1 mark)

$\therefore x = \frac{y/x}{1 + y/x}$ (1 mark)

$\Rightarrow x = \frac{y}{x+y}$ (1 mark)

$\Rightarrow y = \frac{x^2}{1-x}$ (1 mark)

[5 marks]

(c) (i) L: $y - 0 = \frac{0-1}{-2-0}(x+2)$ (2 marks)

$2y = x+2$ (1 mark)

[3 marks]

(ii) M: $y - \frac{11}{2} = -2\left(x - \frac{3}{2}\right)$ (1 mark)

$2y + 4x = 17$ (1 mark)

[2 marks]

(iii) $\dfrac{x+2}{2} = \dfrac{17-4x}{2}$ (1 mark)

$\Rightarrow \quad x = 3, \qquad y = 5/2$ (2 marks)

[3 marks]

Total 25 marks

Specific Objectives: (b) 1, 2, 3, 6,9

SECTION C

(MODULE 3)

Question 5

(a)

$$\lim_{h \to 0} \frac{h}{\sqrt{x+h} - \sqrt{x}} = \lim_{h \to} \frac{h(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})} \quad \text{(1 mark)}$$

$$= \lim_{h \to 0} \frac{h(\sqrt{x+h} + \sqrt{x})}{(x+h) - x} \quad \text{(1 mark)}$$

$$= \frac{\lim}{h \to 0} \frac{h(\sqrt{x+h} + \sqrt{x})}{h} \quad \text{(1 mark)}$$

$$= \lim_{h \to 0} (\sqrt{x+h} + \sqrt{x}) \quad \text{(1 mark)}$$

$$= \sqrt{x} + \sqrt{x} \quad \text{(1 mark)}$$

$$= 2\sqrt{x} \quad \text{(1 mark)}$$

[6 marks]

(b) $f'(x) = 9x^2 + 4x + c$

$\Rightarrow \quad f(x) = 3x^3 + 2x^2 + cx + d$ (1 mark)

Now $f(2) = 14 \Rightarrow \quad 32 + 2c + d = 14$

$\Rightarrow \quad 2c + d = -18$.... I (1 mark)

$f(3) = 74 \Rightarrow \quad 99 + 3c + d = 74$ (1 mark)

$\Rightarrow \quad 3c + d = -25$ ....II (1 mark)

From I + II, $\quad c = -7$ (1 mark)

$d = -4$ (1 mark)

So $f(4) = 3(4^3) + 2(4^2) - 7(4) - 4$

$$= 192 + 32 - 32$$

$$= 192$$ (1 mark)

[7 marks]

(i) $$\frac{dy}{dx} = \frac{(1+x^2)1 - x(2x)}{(1+x^2)^2}$$ (2 marks)

$$= \frac{1-x^2}{(1+x^2)^2}$$ (1 mark)

$$= \frac{1}{(1+x^2)^2} - \frac{x^2}{(1+x^2)^2}$$ (1 mark)

$$= \frac{1}{(1+x^2)^2} - \left(\frac{x}{1+x^2}\right)^2$$ (1 mark)

$$= \frac{1}{(1+x^2)^2} - y^2$$ (1 mark)

[6 marks]

(ii) $$\frac{d^2y}{dx^2} = \frac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2)(2x)}{(1+x^2)^4}$$ (3 marks)

$$= \frac{(1+x^2)(2x)[-(1+x^2) - 2 + 2x^2]}{(1+x^2)^4}$$ (2 marks)

$$= \frac{2y(x^2-3)}{(1+x^2)^2}$$ (1 mark)

[6 marks]

Total 25 marks

Specific Objectives: (a) 3, 4, 5,6; (b) 8, 9,16; (c) 1, 2, 3, 5, 9

Question 6

(a) (i) Finding the equation of the tangent PQ.

a) $\frac{dy}{dx} = 3x^2$ (1 mark)

$\left(\frac{dy}{dx}\right)_{x=3} = 3(3)^2 = 27$ (1 mark)

Equation of tangent:

$y - 27 = 27(x-3)$ (1 mark)

$y = 27x - 54$ (1 mark)

[4 marks]

b) Q has coordinates (2, 0) (1 mark)

[1 mark]

(ii) a) Area $= \int_0^3 dx - \frac{1}{2}(3-2)(27)$ (2 marks)

$= \frac{1}{4}x^4 \Big|_0^3 - \frac{27}{2}$ (2 marks)

$= \frac{81}{4} - \frac{27}{2}$

$= \frac{27}{4}$ units^2 (1 mark)

[5 marks]

b) Required Volume $= \int_0^3 \pi y^2 dx$ - Volume of the cone radius 27 units and height 1 unit. (1 mark)

$= \pi \int_0^3 x^6 dx - \frac{1}{3}\pi(27)^2$ (1 mark)

$= \pi \frac{x^7}{7} \Big|_0^3 - \frac{1}{3}\pi(3^6)$ (1 mark)

$= \pi\left(\frac{3^7}{7}\right) - \frac{1}{3^1}\pi(3^6)$

$= \frac{\pi}{7}\left(3^7 - 7(3^5)\right)$ (1 mark)

$= \frac{\pi}{7}\left(2 \times 3^5\right)$ units^3 (1 mark)

[5 marks]

(b) (i) $\frac{dy}{dx} = 3x^2 - 8x + 5$

$y = x^3 - 4x^2 + 5x + C$ (1 mark)

substituting; $y = 3$ at $x = 0$

$C = 3$ (1 mark)

$y = x^3 - 4x^2 + 5x + 3$ (1 mark)

[3 marks]

(ii) $\frac{dy}{dx} = 0 \Rightarrow 3x^3 - 8x + 5 = 0$

$(3x - 5)(x - 1) = 0$ (2 marks)

$x = \frac{5}{3}; 1$

$y = \frac{131}{27}, 5$ (2 marks)

coordinates are $\left(\frac{5}{3}, \frac{131}{27}\right)$, $(1, 5)$

$\frac{d^2y}{dx^2} = 6x - 8$ (1 mark)

$\left(\frac{d^2y}{dx^2}\right)_{x=5/3} > 0 \qquad \left(\frac{5}{3}, \frac{131}{27}\right)_{max}$ (1 mark)

$\left(\frac{d^2y}{dx^2}\right)_{x=1} < 0 \qquad (1, 5)_{min}$ (1 mark)

[7 marks]

Total 25 marks

Specific Objective(s): (b) 11, 13, 14, 15, 16, 17;
(c) 8 (i), (ii)

02134032/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS

PAPER 03B

SOLUTIONS
&
MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a) (i) $\dfrac{3^{x^2}}{81} = 9^{x+2} \Rightarrow \dfrac{3}{3^4} = 3^{2(x+2)}$ (2 marks)

$\Rightarrow 3^{x^2-4} = 3^{2(x+2)}$ (1 mark)

$\Rightarrow x^2 - 4 = 2(x+2)$ (1 mark)

$\Rightarrow x2 - 2x - 8 = 0$ (1 mark)

$\Rightarrow (x+4)(x+2) = 0$ (1 mark)

$\Rightarrow x = 4 \text{ or } -2.$ (1 mark)

[7 marks]

(ii) $|x+4| = |2x-1| \Rightarrow |x+4|^2 = |2x-1|^2$ (1 mark)

$\Rightarrow (x+4)^2 = (2x-1)^2$ (1 mark)

$\Rightarrow (x+4)^2 - (2x-1)^2 = 0$ * (1 mark)

$\Rightarrow (x+4+2x-1)(x+4-2x+1) = 0$ (2 marks)

$\Rightarrow (3x+3)(5-x) = 0$ (1 mark)

$\Rightarrow x = -1, 5.$ (1 mark)

[7 marks]

Alternatively 1

From * $x^2 + 8x + 16 - 4x^2 + 4x - 1 = 0$ (2 marks)

$\Rightarrow 3x^2 - 12x - 15 = 0$

$\Rightarrow x^2 - 4x - 5 = 0$ (1 mark)

$\Rightarrow (x-5)(x+1) = 0$

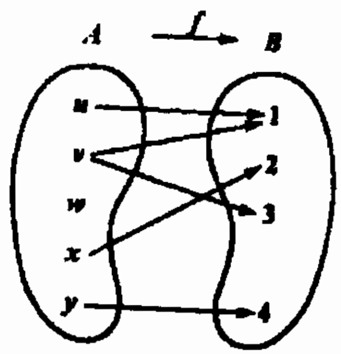
$\Rightarrow x = 5, -1.$ (1 mark)

Alternatively 2

$|1x+4| = |2x-1| \Rightarrow x+4 = 2x-1$ or $x+4 = -2x+1$ (3 marks)

$\Rightarrow x = 5$ or $3x = -3$ (2 marks)

$\Rightarrow x = 5$ or $x = -1$ (2 marks)



(b) (i) f is not a function because f is one-to-many
or because two ordered pairs have the same first element. (1 mark)

(ii) f becomes a functions g: $A \rightarrow B$ if the coach

- assigns only one athlete to Activity 1
- does not assign Activities 1 and 3 to the same athlete.

(2 marks)

(iii) $g = \{(u,1),(v,3),(x,2),(y,4)\}$
Appropriate set notation (1) (3marks)

[6 marks]

Total 20 marks

Specific Objectives: (b) 2,4; (c) 1; (e) 4; (d) 1, 2

SECTION B

(MODULE 2)

Question 2

(a) (i)

d (days)	0	25
w (gms)	500	1500

Both entries correct (1 mark)

[1mark]

(ii) a) Slope $= \dfrac{1500-500}{25-0}$

$= \dfrac{1000}{25}$

$= 40$ (1 mark)

Using point-slope form:

$w - 500 = 40(d - o)$

$\Rightarrow w = 500 + 40d$ (1 mark)

$\therefore f(d) = 40d + 500$ (1 mark)

[3 marks]

b) $w = 500 + 40\,(d)$

$d = 10 \quad = \text{w } 500 + 40\,(10)$ (1 mark)

$= 900$ gms (1 mark)

[2 marks]

(iii) $w = 500 + 40\,(d)$

$\Rightarrow 2180 = 500 + 40\,d$ (1 mark)

$\Rightarrow 1680 = 40\,d$

$\Rightarrow 42 = d$

After 42 days (1 mark)

[2 marks]

(b) (i) $(\tan\theta - \sec\theta)^2 = \tan^2\theta - 2\tan\theta\sec\theta + \sec^2\theta$ (1 mark)

$$= \frac{\sin^2\theta}{\cos^2\theta} - 2\frac{\sin\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}$$ (1 mark)

$$= \frac{\sin^2\theta - 2\sin\theta + 1}{\cos^2\theta}$$ (1 mark)

[3 marks]

Alternatively

$$(\tan\theta - \sec\theta)^2 = \left(\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)^2$$ (1 mark)

$$= \frac{(\sin\theta - 1)^2}{\cos^2\theta}$$ (1 mark)

$$= \frac{\sin^2\theta - 2\sin\theta + 1}{\cos^2\theta}$$ (1 mark)

[3 marks]

(ii) $$\frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$ (1 mark)

$$= \frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$ (1 mark)

$$= \frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$$ (1 mark)

$$\therefore \frac{1-\sin\theta}{1+\sin\theta} = (\tan\theta - \sec\theta)^2$$ (1 mark)

[4 marks]

Alternatively

$$(\tan\theta - \sec\theta)^2 = \frac{(1-\sin\theta)^2}{\cos^2\theta} \quad \text{(1 mark)}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} \quad \text{(1 mark)}$$

$$= \frac{(1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} \quad \text{(1 mark)}$$

$$= \frac{1-\sin\theta}{1-\sin\theta} \quad \text{(1 mark)}$$

[4 marks]

(c) (i) $|OA||OC|\cos A\hat{O}C = 4(1) + 1(7)$ (1 mark)

$$\Rightarrow \cos A\hat{O}C = \frac{11}{|OA||OC|} \quad \text{(1 mark)}$$

$$\Rightarrow \cos A\hat{O}C = \frac{11}{\sqrt{4^2+1^2}\sqrt{1^2+7^2}} \quad \text{(1 mark)}$$

$$\Rightarrow \cos A\hat{O}C = \frac{11}{\sqrt{17}\sqrt{50}} = \frac{11}{29.15}$$

$$\Rightarrow \cos A\hat{O}C = 0.377 \quad \text{(1 mark)}$$

[4 marks]

(ii) $A\hat{O}C = 67.8°$ (1 mark)

[1 mark]

Total 20 marks

Specific Objectives: (a) 6, 7, 8, 9, 10;
(b) 1, 3; (c) 9, 10

SECTION C

(MODULE 3)

Question 3

(a) (i) $$\lim_{x \to 4} \frac{\sqrt{x-2}}{x-4} = \lim_{x \to 4} \frac{\sqrt{x-2}}{\left(\sqrt{x-2}\right)\left(\sqrt{x+2}\right)}$$ (1 mark)

$$= \lim_{x \to 4} \frac{1}{\sqrt{x+2}}$$ (1 mark)

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$ (1 mark)

[3 marks]

(ii) $$\lim_{x \to 4} \frac{\sqrt{x-2}}{x^2-5x+4} = \lim_{x \to 4} \frac{\sqrt{x-2}}{(x-4)(x-1)}$$ (1 mark)

$$= \lim_{x \to 4} \frac{\sqrt{x-2}}{x-4} \cdot \lim_{x \to 4} \frac{1}{x-1}$$ (1 mark)

$$= \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{1}{12}$$ (1 mark)

[3 marks]

(b) $$y = \frac{A}{x} + Bx \Rightarrow \frac{dy}{dx} = \frac{A}{x^2} + B$$ (2 marks)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2A}{x^3}$$ (1 mark)

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{2A}{x} - \frac{A}{x} + Bx$$ (1 mark)

$$= \frac{A}{x} + Bx$$

$$= y$$ (1 mark)

[5 marks]

(c) (i) Let h be height t hr after the flooding started.

$\frac{dh}{dt} = 14 - 13t$ (1 mark)

$\frac{dh}{dt} = 14 - 13t \Rightarrow h = 14t - \frac{3t^2}{2} + \text{constant}$ (1 mark)

$h = o$ at $t = o \Rightarrow h = 14t - \frac{3t^2}{2}$ (1 mark)

$\frac{dh}{dt} = o \qquad \Rightarrow t = \frac{14}{3}$ (1 mark)

$\frac{d^2h}{dt^2} = -3$ so at $t = \frac{14}{3}$, h max occurs. (1 mark)

[5 marks]

(ii) $h_{\max} = 14\frac{14}{3} - \left(\frac{3}{2}\right)\left(\frac{14}{3}\right)^2$

$= \frac{196}{6}$

$= 32\ \frac{2}{3}$ inches (1 mark)

[1 mark]

(iii) At the step, h= 2ft 6 in.

So $30 = 14t - \frac{3}{2}t^2$ (1 mark)

$\Rightarrow \quad 3t^2 - 28t + 60 = 0$

$\Rightarrow \quad (3t - 10)(t - 6) = 0$ (1 mark)

$\Rightarrow \quad t = \frac{10}{3}$ or $t = 6$

$\Rightarrow \quad t = 3\ \frac{1}{3}$ (1 mark)

[3 marks]

Total 20 marks

Specific Objectives: (a) 3, 4, 6;
(b) 5, 6, 7,16;
(c) 1, 3,4,5,9