FORM TP 2012037

TEST CODE **01254020**

MAY/JUNE 2012

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

03 MAY 2012 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. DO NOT open this examination paper until instructed to do so.
2. This paper consists of FOUR sections. Answer ALL questions in Section I, Section II and Section III.
3. Answer ONE question in Section IV.
4. Write your solutions with full working in the booklet provided.

2037

Required Examination Materials

Electronic Calculator (non programmable)
Geometry Set
Mathematical Tables (provided)
Graph Paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION I

Answer BOTH questions.

1. (a) The functions f and g are defined by

$$f(x) = x^3 + 1, \qquad 0 \le x \le 3$$

$$g(x) = x + 5, \qquad x \in \mathbf{R}$$

where **R** is the set of real numbers.

(i) Determine the composition function $g(f(x))$. **(1 mark)**

(ii) State the range of $g(f(x))$. **(1 mark)**

(iii) Determine the inverse of $g(f(x))$. **(2 marks)**

(b) If $x + 2$ is a factor of $f(x) = 2x^3 - 3x^2 - 4x + a$, find the value of a. **(2 marks)**

(c) Solve the equation $3^{2x} - 9(3^{-2x}) = 8$. **(5 marks)**

(d) (i) Express $x^3 = 10^{x-3}$ in the form $\log_{10}x = ax + b$. **(2 marks)**

(ii) Hence, state the value of the gradient of a graph of $\log_{10}x$ versus x. **(1 mark)**

Total 14 marks

2. (a) The quadratic equation $x^2 - 4x + 6 = 0$ has roots α and β.

Calculate the value of $\alpha^2 + \beta^2$. **(5 marks)**

(b) Find the range of values of x for which

$$\frac{2x-5}{3x+1} > 0.$$ **(4 marks)**

(c) A customer repays a loan monthly by increasing the payment each month by \$$x$. If the customer repaid \$50 in the 5th month and \$70 in the 9th month, calculate the TOTAL amount of money repaid at the end of the 24th month. **(5 marks)**

Total 14 marks

SECTION II

Answer BOTH questions.

3. (a) The equation of a circle is given by $x^2 + y^2 - 4x + 6y = 87$.

(i) A line has equation $x + y + 1 = 0$. Show that this line passes through the centre of the circle. **(3 marks)**

(ii) Find the equation of the tangent to the circle at the point $A\ (-6, 3)$. **(4 marks)**

(b) Given $\overrightarrow{\mathbf{OA}} = \mathbf{a}$, $\overrightarrow{\mathbf{OB}} = \mathbf{b}$, $\overrightarrow{\mathbf{AP}} = \frac{1}{2}\overrightarrow{\mathbf{OA}}$,

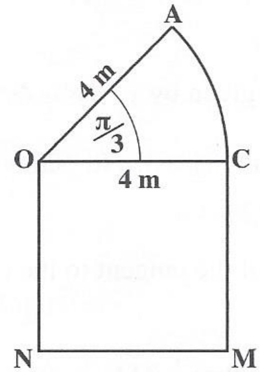
where $\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

(i) Write $\overrightarrow{\mathbf{BP}}$ in terms of **a** and **b**. **(2 marks)**

(ii) Find $|\overrightarrow{\mathbf{BP}}|$. **(3 marks)**

Total 12 marks

4. (a) The diagram shows a sector of a circle centre O with an adjoining square. The radius of the circle is 4 m.



If the sector AOC subtends an angle $\frac{\pi}{3}$ at O, calculate, **giving your answer in terms of** $\boldsymbol{\pi}$

(i) the area of the shape $OACMN$

(ii) the perimeter of the shape $OACMN$. **(5 marks)**

(b) Given that $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, evaluate without using calculators, the **exact** value of $\cos\frac{7\pi}{12}$. **(3 marks)**

(c) Prove the identity

$$\frac{1}{\sec\theta + \tan\theta} \equiv \frac{1 - \sin\theta}{\cos\theta}.$$ **(4 marks)**

Total 12 marks

SECTION III

Answer BOTH questions.

5. (a) Differentiate the following expression with respect to x, simplifying your answer.

$$\frac{3x + 4}{x - 2}$$

(4 marks)

(b) The point P (2, 10) lies on the curve $y = 3x^2 + 5x - 12$. Find equations for

(i) the tangent to the curve at P

(ii) the normal to the curve at P. **(5 marks)**

(c) The length of the side of a square is increasing at a rate of 4 cms^{-1}. Find the rate of increase of the area when the length of the side is 5 cm. **(5 marks)**

Total 14 marks

6. (a) Evaluate $\int_1^2 (16 - 7x)^3 \, dx$. **(4 marks)**

(b) The point Q (4, 8) lies on a curve for which $\frac{dy}{dx} = 3x - 5$. Determine the equation of the curve. **(3 marks)**

(c) Calculate the area between the curve $y = 2 \cos x + 3 \sin x$ and the x-axis from $x = 0$ to $\frac{\pi}{3}$.

(3 marks)

(d) Calculate the volume of the solid formed when the area enclosed by the curve $y = x^2 + 2$ and the x-axis, from $x = 0$ to $x = 3$, is rotated through 360° about the x-axis.

[Leave your solution in terms of π]. **(4 marks)**

Total 14 marks

SECTION IV

Answer only ONE question.

7. (a) A survey carried out in a town revealed that 25% of the households surveyed owned a laptop computer and 70% owned a desktop computer. In addition, it was found that 12% owned both a laptop and a desktop computer.

If a sample of households from the town is selected at random, determine the proportion that own NEITHER a laptop NOR a desktop computer.

(4 marks)

(b) A bag contains 4 red marbles, 3 black marbles and 3 blue marbles. Three marbles are drawn at random without replacement from the bag.

Find the probability that the marbles

(i) drawn are ALL of the SAME colour **(3 marks)**

(ii) contain EXACTLY 1 red marble. **(3 marks)**

(c) The probability of hiring a taxi from garage A, B or C is 0.3, 0.5 and 0.2 respectively. The probability that the taxi ordered will be late from A is 0.07, from B is 0.1 and from C is 0.2.

(i) Illustrate this information on a tree diagram showing the probability on all branches. **(3 marks)**

(ii) A garage is chosen at random, determine the probability that

a) the taxi will arrive late **(3 marks)**

b) the taxi will come from garage C given that it is late. **(4 marks)**

Total 20 marks

8. (a) A car starting from rest at a point A, moves along a straight line reaching a velocity of 24 ms^{-1} by a constant acceleration of 6 ms^{-2}. The car maintains this constant velocity of 24 ms^{-1} for 5 seconds and is then brought to rest again by a constant acceleration of -3 ms^{-2}.

(i) **Using the graph sheet provided**, draw a velocity-time graph to illustrate the motion of the car. **(3 marks)**

(ii) Determine the TOTAL distance travelled by the car. **(3 marks)**

(iii) A second car, moving at a constant velocity of 32 ms^{-1} drives past point A, 3 seconds after the first car left point A. Calculate the length of time after the first car started that this second car meets it.

[Assume that the cars meet during the time when the first car is moving at a constant velocity.] **(4 marks)**

(b) A particle moves in a straight line with acceleration given by $a = (5t - 1)$ ms^{-2} at any time t seconds. When $t = 2$ seconds, the particle has velocity 4 ms^{-1} and is 8 m from a fixed point O. Determine

(i) its velocity when $t = 4$ **(5 marks)**

(ii) its displacement from O when $t = 3$. **(5 marks)**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.